Answer key Review Final Exam MAC1114 (1)

(1) a) 1 degree $= \frac{\pi}{180}$ rad so $\boxed{330° = \frac{\pi\, 330}{180} = \frac{11}{6}\pi \text{ rad}}$

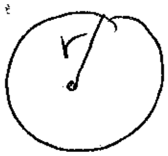
$\boxed{-225° = -\frac{\pi}{180}(225) = -\frac{5}{4}\pi \text{ rad}}$

b) $\pi = 180$ degrees or $1\text{ rad} = \frac{180}{\pi}$

$\theta = -\frac{2\pi}{3} = -\frac{2(180)}{3} = -120°$

$\theta = 3 = \frac{3(180)}{\pi} = \frac{540}{\pi}$ degrees

Careful π here is 3.1415---

c)  You must convert θ to radians

$\theta = 120° = \frac{120\pi}{180} = \frac{2\pi}{3}$ $s = r\theta = 3\left(\frac{2\pi}{3}\right) = 2\pi$ meters

$s \approx 6.28$

d) Must convert θ to radians

$\theta = 36° = \frac{36\pi}{180} = \frac{\pi}{5}$ $\boxed{\text{Area} = \frac{1}{2}r^2\theta = \frac{1}{2}(6)^2\left(\frac{\pi}{5}\right) = \frac{18\pi}{5} \text{ square feet}}$

e) From the formula $s = r\theta$, $3 = 5\theta$ so $\theta = \frac{3}{5}$ radians

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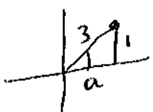
(2) acute Quad I  $a^2+1=3^2$, $a^2 = 8$, but $a>0$, $a = \sqrt{8} = 2\sqrt{2}$

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{\sqrt{8}}{3}$ $\tan\theta = \frac{1}{\sqrt{8}} = \frac{\sqrt{8}}{8} = \frac{\sqrt{2}}{4}$

$\cot\theta = \sqrt{8}$, $\sec\theta = \frac{3\sqrt{8}}{8} = \frac{3\sqrt{2}}{4}$, $\csc\theta = 3$

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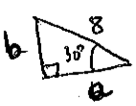
(3)  $\sin 30° = \frac{b}{8}$ so $b = 8\sin 30° = 4$ in, $a = 8\cos 30° = 4\sqrt{3}$ inches

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(4) a) $= 6(1) - 8\left(\frac{1}{2}\right) = 6 - 4 = 2$ b) $= \cot 40° - \frac{\cos(40°)}{\sin 40°} = \cot 40° - \cot 40° = 0$

c) $= 1 - \cos^2 20° - \sin^2 20° = 1 - (\cos^2 20° + \sin^2 20°) = 1 - 1 = 0$

d) $= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{1}{2}$ e) $1 + \tan^2 30° - \csc^2 45° = 1 + \frac{1}{3} - 2 = -\frac{2}{3}$

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(5)

$\sin>0$	$\sin>0$
$\sin<0$ $\rightarrow \tan>0$	$\sin<0$

Answer Quadrant III

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6) a) ref of $-20° = 20°$ b) ref $240° = 60°$ c) ref $= 60°$ d) $70°$

e) $\frac{2\pi}{7}$

Answerkey Review Final Exam MAC1114 (2)

(7) ref $= 30°$, Quad III $\cos < 0$ $\cos 210° = -\cos 30° = -\frac{\sqrt{3}}{2}$

ref $= 60°$ Quad II $\tan < 0$ $\tan 120° = -\tan 60° = -\sqrt{3}$

ref $= 60°$ Quad I > 0 $\sec 420° = +\sec 60° = 2$

ref $= \frac{\pi}{4}$ Quad I > 0 $\sin \frac{9\pi}{4} = +\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

ref $= 60°$ Quad II > 0 $\sin(-240°) = +\sin 60° = \frac{\sqrt{3}}{2}$

ref $= \frac{\pi}{3}$ Quad II < 0 $\tan\left(\frac{14\pi}{3}\right) = -\tan\left(\frac{\pi}{3}\right) = -\sqrt{3}$

ref $= \frac{\pi}{6}$ Quad IV < 0 $\cot\left(-\frac{\pi}{6}\right) = -\cot\left(\frac{\pi}{6}\right) = -\sqrt{3}$

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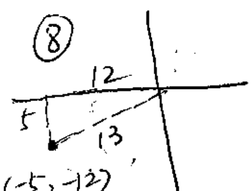
(8)



$a^2 + 25 = 13^2 = 169$

$a < 0$ Quad III

$a^2 = 169 - 25 = 144$

$a = -12$

let $b = $ opp $= -5$

$\cos\theta = \frac{a}{13} = \frac{-12}{13}$, $\tan\theta = \frac{b}{a} = \frac{5}{12}$, $\cot\theta = \frac{a}{b} = \frac{12}{5}$

$\sec\theta = \frac{1}{\cos\theta} = -\frac{13}{12}$, $\csc\theta = \frac{1}{\sin\theta} = -\frac{13}{5}$

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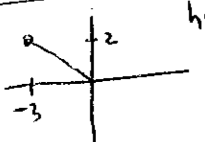
(9)



$hyp^2 = 9 + 4 = 13$

$hyp = \sqrt{13}$

Identities $\cot\theta = \frac{1}{\tan\theta} = -\frac{3}{2}$

$\sec^2\theta = \tan^2\theta + 1 = \frac{13}{9}$

Quad II $\sec\theta < 0$, $\sec\theta = -\frac{\sqrt{13}}{3}$

$\cos\theta = \frac{1}{\sec\theta} = -\frac{3}{\sqrt{13}}$

$\sin\theta = \cos\theta\tan\theta$

$\sin\theta = -\frac{3}{\sqrt{13}} \cdot \frac{-2}{3} = \frac{2}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$

$\csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{13}}{2}$

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(10) Domain $\sin = (-\infty, \infty)$, Domain cosine $= (-\infty, \infty)$ Range $\cos = [-1, 1]$

Range $\sin = [-1, 1]$

etc....

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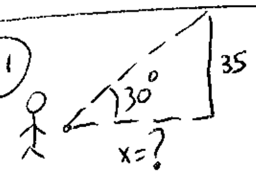
(11)



$\tan 30° = \frac{35}{x}$ so $x = \frac{35}{\tan 30°} = 35\sqrt{3}$ feet ← Answer

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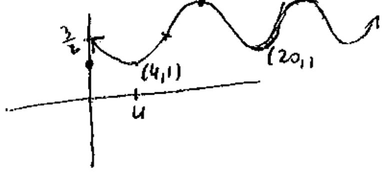
(12) Period $T = \frac{2\pi}{\frac{\pi}{8}} = 16$ Amplitude $= \left|-\frac{1}{2}\right| = \frac{1}{2}$ 5 key points 0 4 8 12 16

Key points are $(0, f(0)) = (0, \frac{3}{2})$, $(4, 1)$, $(8, \frac{3}{2})$, $(12, 2)$, $(16, \frac{3}{2})$



Answerkey Review Final Exam MAC1114 (3)

(13) ~~review~~ period $\tan(\omega x) = \frac{\pi}{\omega}$ a) period $\tan\left(\frac{x}{2}\right) = \frac{\pi}{1/2} = 2\pi$

b) period $\cot(\mu x) = \frac{\pi}{\mu}$ so period $-1+2\cot(x) = \pi$

c) period $\csc\left(\frac{3\pi x}{2}\right) = \frac{2\pi}{\frac{3\pi}{2}} = \frac{4}{3}$ d) period $1+3\sec\left(\frac{x}{4}\right) = \frac{2\pi}{\frac{1}{4}} = 8\pi$

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(14) notes or book

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(15) a) $-\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) $-\frac{\pi}{6}$ d) $\frac{2\pi}{3}$ $\cos^{-1}(-1) = \pi$

e) $\frac{3\pi}{4}$ f) $\frac{\pi}{4}$ g) $\frac{5\pi}{6}$ h) $-\frac{\pi}{3}$ $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

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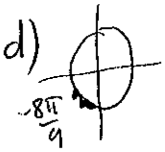
(16) careful $\sin^{-1}\sin\theta = \theta$ only if $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$, $\cos^{-1}\cos\theta = \theta$ only if $0 \le \theta \le \pi$

$\tan^{-1}\tan\theta = \theta$ only if $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $\cot^{-1}\cot\theta = \theta$ only if $0 < \theta < \pi$

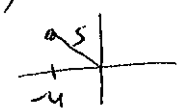
a) $0 < \frac{3\pi}{8} < \frac{\pi}{2}$ so Ans $= \frac{3\pi}{8}$ b) $0 \le \frac{3\pi}{4} \le \pi$ so Answer $= \frac{3\pi}{4}$ c) Answer $= -\frac{2\pi}{5}$

d) 

not in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ ref $= \frac{\pi}{9}$ so $\sin^{-1}\sin\left(-\frac{8\pi}{9}\right) = \sin^{-1}\sin\left(-\frac{\pi}{9}\right) = -\frac{\pi}{9}$ d)

d2) $\sin(\sin^{-1}x) = x$ if $-1 \le x \le 1$ so Ans ge $= -0.7$ e) D.N.E $\sqrt{2} > 1$ undefined

f) $\theta = \cos^{-1}\left(-\frac{4}{5}\right)$ if θ in Quad II, $\cos\theta = -\frac{4}{5}$ $\tan\left(\cos^{-1}\left(-\frac{4}{5}\right)\right) = \tan\theta = \frac{opp}{adj} = -\frac{3}{4}$



opp = 3
adj = -4

g) Answer $= \frac{4}{5}$ because $\sin\theta = \frac{3}{5}$ h) Answer $= \frac{4}{5}$ skip $\cos\tan^{-1}\left(\frac{4\pi}{3}\right)$!

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(18) a) Quad II $\cos\alpha = -\frac{3}{5}$, Quad III $\cos\beta = -\frac{1}{\sqrt{5}}$ $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha$

$= \frac{-4}{5\sqrt{5}} + \frac{6}{5\sqrt{5}} = \frac{+2}{5\sqrt{5}} = \frac{2\sqrt{5}}{25}$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta = \frac{3}{5\sqrt{5}} - \frac{(-8)}{5\sqrt{5}} = \frac{11\sqrt{5}}{25}$

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b) $\cos\alpha = \frac{-12}{13}$, $\cos\beta = -\frac{1}{2}$, $\sin\beta = \frac{\sqrt{3}}{2}$ $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \frac{-\frac{5}{12} - \sqrt{3}}{1 - \left(\frac{5\sqrt{3}}{12}\right)} = \frac{-5-12\sqrt{3}}{12-5\sqrt{3}}$

$\tan\alpha = -\frac{5}{12}$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha = \frac{-120}{169}$

Answerkey Review Final Exam MAC1114 (4)

(18) c) $\cos\theta = -\frac{\sqrt{8}}{3} = \frac{-2\sqrt{2}}{3}$, $\sin(\theta+\frac{\pi}{6}) = \sin\theta\cos\frac{\pi}{6} + \cos\theta\sin\frac{\pi}{6} = \frac{\sqrt{3}}{6} - \frac{2\sqrt{2}}{6} = \frac{\sqrt{3}-2\sqrt{2}}{6}$

$\cos(\theta-\frac{\pi}{3}) = \cos\theta\cos\frac{\pi}{3} + \sin\theta\sin\frac{\pi}{3} = \frac{-2\sqrt{2}}{6} + \frac{\sqrt{3}}{6} = \frac{-2\sqrt{2}+\sqrt{3}}{6}$

$\tan(\theta+\frac{\pi}{4}) = \frac{\tan\theta + \tan\frac{\pi}{4}}{1-\tan\theta\tan\frac{\pi}{4}} = \frac{-\frac{\sqrt{2}}{4}+1}{1+\frac{\sqrt{2}}{4}} = \frac{-\sqrt{2}+4}{4+\sqrt{2}} = \tan(\theta+\frac{\pi}{4})$

(18) d) ~~First~~ sum formula $\sin(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{1}{2}) = \sin(\sin^{-1}\frac{3}{5})\cos(\cos^{-1}\frac{1}{2}) + \sin(\cos^{-1}\frac{1}{2})\cos\sin^{-1}\frac{3}{5}$

so

$\sin(\sin^{-1}\frac{3}{5} + \cos^{-1}\frac{1}{2}) = (\frac{3}{5})(\frac{1}{2}) + \frac{\sqrt{3}}{2}(\frac{4}{5}) = \frac{3+4\sqrt{3}}{10}$

next $\sin(\sin^{-1}\frac{3}{5} - \cos^{-1}(-\frac{4}{5})) = \underbrace{\sin(\sin^{-1}\frac{3}{5})}\underbrace{\cos(\cos^{-1}(-\frac{4}{5}))} - \underbrace{\sin\cos^{-1}(-\frac{4}{5})}\cos(\sin^{-1}\frac{3}{5})$

$= \frac{3}{5}(-\frac{4}{5}) - (\frac{3}{5})(\frac{4}{5}) = -\frac{24}{25}$

(19) a) $\sin\theta = \frac{1}{2}$ Quad I, II Ans. $\frac{\pi}{6}, \frac{5\pi}{6}$ ~~b)~~ b) $\cos(3\theta) = -\frac{\sqrt{3}}{2}$

b) General solutions first $3\theta = \frac{5\pi}{6} + 2k\pi$ or $3\theta = \frac{7\pi}{6} + 2k\pi$

divide by ~~3~~ 3 to get $\theta = \frac{5\pi}{18} + \frac{2k\pi}{3}$ or $\theta = \frac{7\pi}{18} + \frac{2}{3}k\pi$

List Answers on $[0,2\pi)$: $\frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$

c) double angle $2\cos^2\theta - 1 + \cos\theta = 0$ Factor $(2\cos\theta-1)(\cos\theta+1) = 0$

$\cos\theta = \frac{1}{2}$, $\cos\theta = -1$

Answers $\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$

d) General solutions, $2\theta = \frac{\pi}{4} + k\pi$ divide by 2, $\theta = \frac{\pi}{8} + \frac{k\pi}{2}$

Answer $\{\frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}\}$

e) double angle and factor $2\sin\theta\cos\theta = \sin\theta$ so $\sin\theta(2\cos\theta-1) = 0$

$\sin\theta = 0$, $\cos\theta = \frac{1}{2}$

Answer $\{0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}\}$

Answer key Review Final Exam MAC1114 (5)

(19) f) General solutions: $\frac{\theta}{2} = \pi + 2k\pi$ so $\theta = 2\pi + 4k\pi$

No solutions in $[0, 2\pi)$

g) Factor $4\sin^2\theta - 1 = (2\sin\theta - 1)(2\sin\theta + 1) = 0$ if $\sin\theta = \pm\frac{1}{2}$ all quadrants

Answer $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$

h) General solutions $3\theta = \frac{\pi}{3} + k\pi$ so $\theta = \frac{\pi}{9} + \frac{k\pi}{3}$

Answer $\left\{ \frac{\pi}{9}, \frac{4\pi}{9}, \frac{7\pi}{9}, \frac{10\pi}{9}, \frac{13\pi}{9}, \frac{16\pi}{9} \right\}$

i) Factor $4\cos^2\theta - 3 = (2\cos\theta - \sqrt{3})(2\cos\theta + \sqrt{3})$ so $\cos\theta = \pm\frac{\sqrt{3}}{2}$

all quadrants

Answer $\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6} \right)$

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(20) quad III, $\sin\theta < 0$, $\sin\theta = -\frac{4}{5}$ a) $\sin 2\theta = 2\sin\theta\cos\theta = \frac{24}{25}$ b) $\cos(2\theta) = 2\cos^2\theta - 1$ $= -\frac{7}{25}$ b)

c) $\tan 2\theta = \frac{\sin 2\theta}{\cos(2\theta)} = -\frac{24}{7}$

d) $\pi < \theta < \frac{3\pi}{2}$ so $\frac{\pi}{2} < \frac{\theta}{2} < \frac{3\pi}{4}$ that means $\frac{\theta}{2}$ is in Quad II $\sin > 0$, $\cos < 0$

$\sin\frac{\theta}{2} = +\sqrt{\frac{1-\cos\theta}{2}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$ e) $\cos\frac{\theta}{2} = -\sqrt{\frac{1+\cos\theta}{2}} = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$

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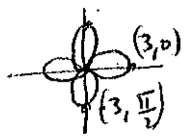
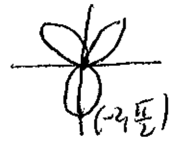
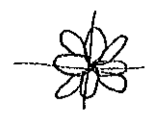
(21) a) rose four petals



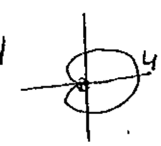
b) rose three petals



c) rose 8 petals

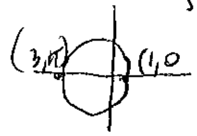


d) cardioid

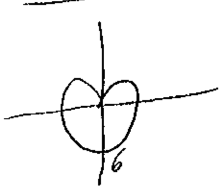


g) limaçon no loops

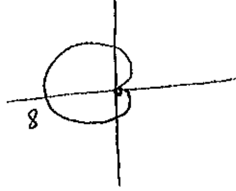
h) limaçon no loops



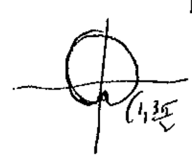
e) cardioid



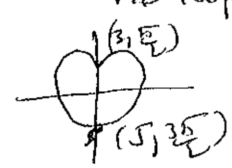
f) cardioid



i) limaçon no loop



j) limaçon no loops



Answer key Review Final exam MAC1114 (6)

(22) a) $x = \frac{-4 - \sqrt{16-32}}{2} = \frac{-4 \pm 4i}{2} = -2 \pm 2i$

b) $(x-2)(x^2+2x+4)=0$

$x=2, \quad x = \frac{-2 \pm \sqrt{4-16}}{2} = -1 \pm 2\sqrt{3}\, i$

Ans $\{2, -1 \pm 2\sqrt{3}i\}$

c) $(x^2-1)(x^2+1)=0$

$x^2=1, \quad x^2=-1$

$x=\pm 1 \quad x=\pm i$

Answer $\{\pm 1, \pm i\}$

(23) $|z| = \sqrt{1+3} = 2, \quad \theta = \frac{\pi}{3} + \pi = \frac{4\pi}{3}$ a) $z = 2\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

b) De Moivre's Theorem $z^5 = 2^5\left(\cos\left(\frac{20\pi}{3}\right) + i\sin\left(\frac{20\pi}{3}\right)\right)$

$= 32\left(\cos\left(\frac{2\pi}{3}\right) + i\sin\left(\frac{2\pi}{3}\right)\right)$

$= 32\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -16 + i16\sqrt{3}$

(c) De Moivre's

$\sqrt[3]{z} = \sqrt[3]{2}\left(\cos\left(\frac{4\pi}{3} + 2k\pi\right) + i\sin\left(\frac{4\pi}{3} + 2k\pi\right)\right)^{1/3}$

$\sqrt[3]{z} = \sqrt[3]{2}\left(\cos\left(\frac{4\pi}{9} + \frac{2}{3}k\pi\right) + i\sin\left(\frac{4\pi}{9} + \frac{2}{3}k\pi\right)\right), \quad k=0,1,2$

There are 3 different cube roots!

(24) $|z| = 8\sqrt{2}, \quad \tan\theta = 1, \quad \theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$, a) $z = 8\sqrt{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

b) $z^5 = (8\sqrt{2})^5\left(\cos\left(\frac{25\pi}{4}\right) + i\sin\left(\frac{25\pi}{4}\right)\right)$, use $\frac{25\pi}{4} = 6\pi + \frac{\pi}{4}$, $\cos\left(\frac{25\pi}{4}\right) = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$

$= (8\sqrt{2})^5\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)$

c) $\sqrt[3]{z} = \left(8\sqrt{2}\cos\left(\frac{5\pi}{4} + 2k\pi\right) + i\sin\left(\frac{5\pi}{4} + 2k\pi\right)\right)^{1/3}$

$= 2\sqrt[6]{2}\left(\cos\left(\frac{5\pi}{12} + \frac{2}{3}k\pi\right) + i\sin\left(\frac{5\pi}{12} + \frac{2}{3}k\pi\right)\right)$ with $k=0,1,2$

Three different cube roots

$z_1 = 2\sqrt[6]{2}(\cos 75^\circ + i\sin 75^\circ)$, $z_2 = 2\sqrt[6]{2}(\cos 195^\circ + i\sin 195^\circ)$

$z_3 = 2\sqrt[6]{2}(\cos 315^\circ + i\sin(315^\circ))$

Answer key Review Final Exam MAC1114 (7)

(25) a) $v = \langle 3, -5 \rangle$, $w = \langle -2, 3 \rangle$ $\quad 5v + 4w = \langle 15, -25 \rangle + \langle -8, 12 \rangle = \langle 15-8, -25+12 \rangle$

$5v + 4w = \langle 7, -13 \rangle = 7\hat{i} - 13\hat{j}$

$\|v\| = \sqrt{9+25} = \sqrt{34}$

$\|w\| = \sqrt{4+9} = \sqrt{13}$, $\quad 5\|v\| + 4\|w\| = 5\sqrt{34} + 4\sqrt{13}$

b) $v + w = \langle 3 + (-2), -5 + 3 \rangle = \langle 1, -2 \rangle$, $\quad \|v + w\| = \sqrt{1+4} = \sqrt{5}$

c)  $\tan\theta = \frac{-5}{3}$, $\theta = \tan^{-1}\left(\frac{-5}{3}\right)$

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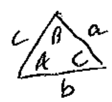
(26) Law of cosins $\quad c^2 = a^2 + b^2 - 2(ab)\cos C = 36 + 16 - 48\cos(60°)$



$c^2 = 52 - 48\left(\frac{1}{2}\right) = 28$

$c = \sqrt{28}$ or $2\sqrt{7} = c$

$a^2 = b^2 + c^2 - 2bc\cos A$

solve $\quad \cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{8}{2(8)\sqrt{7}} = \dfrac{1}{2\sqrt{7}} = \dfrac{1}{2\sqrt{7}}$

$A = \cos^{-1}\left(\frac{1}{2\sqrt{7}}\right)$ $\quad B = 180° - 60° - A$

---

(27) a) $\sin(195°) = \sin(150° + 45°) = \sin 150° \cos 45° + \cos 150° \sin 45° = \sin 30° \cos 45° - \cos 30° \sin 45°$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}(1-\sqrt{3})}{4}$ or $-\frac{\sqrt{2-\sqrt{3}}}{2}$

b) $\cos(165°) = \cos(120° + 45°) = \cos(\overset{-\frac{1}{2}}{120°})\overset{\frac{\sqrt{2}}{2}}{\cos 45°} - \overset{\frac{\sqrt{3}}{2}}{\sin 120°}\overset{\frac{\sqrt{2}}{2}}{\sin 45°} = \frac{-\sqrt{2} - \sqrt{6}}{4} = \frac{-(\sqrt{2}+\sqrt{6})}{4}$

c) $\tan(195°) = \tan(150° + 45°) = \dfrac{\tan 150° + \tan 45°}{1 - \tan 150° \tan 45°} = \dfrac{\frac{-1}{\sqrt{3}} + 1}{1 - \frac{-1}{\sqrt{3}}} = \dfrac{-1+\sqrt{3}}{1+\sqrt{3}}$

d) half-angle $\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$ so $\cos\left(\frac{\pi}{12}\right) = + \sqrt{\frac{1 + \cos\frac{\pi}{6}}{2}} = \frac{\sqrt{2+\sqrt{3}}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

quad I

quad IV

e) $\sin\left(\frac{\theta}{2}\right) = -\sqrt{\frac{1-\cos\theta}{2}}$, $\quad \sin(-22.5°) = -\frac{\sqrt{2-\sqrt{2}}}{2}$

f) $\cos(-22.5°) = \cos(22.5°) = +\sqrt{\frac{1+\cos 45°}{2}} = \boxed{\frac{\sqrt{2+\sqrt{2}}}{2} = \cos(22.5°)}$

g) $\sec\left(\frac{15\pi}{8}\right) = \frac{1}{\cos\left(\frac{15\pi}{8}\right)}$

$\boxed{\sec\left(\frac{15\pi}{8}\right) = \frac{2}{\sqrt{2+\sqrt{2}}}}$

h) $\sin\left(2\cos^{-1}\left(\frac{4}{5}\right)\right) = 2\sin\left(\cos^{-1}\frac{4}{5}\right)\cos\left(\cos^{-1}\frac{4}{5}\right) = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$ h)

i) $\cos\left(2\sin^{-1}\left(-\frac{2}{5}\right)\right) = 1 - 2\sin^2\left(\sin^{-1}\left(-\frac{2}{5}\right)\right) = 1 - \frac{8}{25} = \frac{17}{25}$

j) $= 2\sin\left(\frac{75°+15°}{2}\right)\cos\left(\frac{75°-15°}{2}\right) = \frac{\sqrt{6}}{2}$

Answerkey Review Final exam MAC1114 (8)

(28) use fractions' rule $\frac{a}{b}+\frac{c}{d}=\frac{ad+bc}{bd}$

a) $\frac{1+\cos\theta}{\sin\theta}+\frac{\sin\theta}{1+\cos\theta}=\frac{(1+\cos\theta)(1+\cos\theta)+\sin\theta\sin\theta}{\sin\theta(1+\cos\theta)}$

identity $=1$

$=\frac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$

cancel out

$=\frac{2+2\cos\theta}{\sin\theta(1+\cos\theta)}=\frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$

$=\frac{2}{\sin\theta}=2\csc\theta$

---

b) $\frac{\tan\theta+\cot\theta}{\sec\theta\csc\theta}=\frac{\frac{\sin\theta}{\cos\theta}+\frac{\cos\theta}{\sin\theta}}{\sec\theta\csc\theta}=$

identity $\to 1$

$=\frac{\sin^2\theta+\cos^2\theta}{(\cos\theta\sin\theta)}\times\frac{1}{\sec\theta\csc\theta}=\frac{1}{\underbrace{(\cos\theta\sec\theta)}_{=1}\underbrace{(\sin\theta\csc\theta)}_{=1}}=1$

---

c) $\frac{1-\sin\theta}{\cos\theta}=\frac{1-\sin\theta}{\cos\theta}\cdot\frac{1+\sin\theta}{1+\sin\theta}=\frac{(1-\sin\theta)(1+\sin\theta)}{(\cos\theta)(1+\sin\theta)}=\frac{1-\sin\theta+\sin\theta-\sin^2\theta}{\cos\theta(1+\sin\theta)}$

$=\frac{1-\sin^2\theta}{\cos\theta(1+\sin\theta)}$

Next use $\cos^2\theta=1-\sin^2\theta$ so

$\frac{1-\sin\theta}{\cos\theta}=\frac{\cos^2\theta}{\cos\theta(1+\sin\theta)}=\frac{\cos\theta}{1+\sin\theta}$ (by cancelling out $\cos\theta$)!

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d) Use $\sec^2\theta=1+\tan^2\theta$

$\frac{\sec^2\theta-\tan^2\theta+\tan\theta}{\sec\theta}=\frac{1+\tan^2\theta-\tan^2\theta+\tan\theta}{\sec\theta}=\frac{1+\tan\theta}{\sec\theta}=\frac{1+\frac{\sin\theta}{\cos\theta}}{\frac{1}{\cos\theta}}$

$=\left(1+\frac{\sin\theta}{\cos\theta}\right)\frac{\cos\theta}{1}$

$=\cos\theta+\sin\theta$

---

f) FACTOR $\cos^4\theta-\sin^4\theta=\underbrace{(\cos^2\theta-\sin^2\theta)}_{\cos(2\theta)}\underbrace{(\cos^2\theta+\sin^2\theta)}_{=1}=\cos(2\theta)$

Answer key Review Final Exam MAC 1114 (9)

(28) e) $$\frac{\cot\theta-\tan\theta}{\cot\theta+\tan\theta}=\frac{\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}}=\frac{\cos^2\theta-\sin^2\theta}{\cos\theta\sin\theta}\cdot\frac{\cos\theta\sin\theta}{\underbrace{\cos^2\theta+\sin^2\theta}_{=1}}$$

$$=\cos^2\theta-\sin^2\theta$$

$$=\cos(2\theta)$$ trig identity.

g) $$\frac{\sin 3\theta}{\sin\theta}-\frac{\cos 3\theta}{\cos\theta}=\frac{\sin 3\theta\cos\theta-\sin\theta\cos 3\theta}{\sin\theta\cos\theta}\xrightarrow{\text{difference formula}}=\frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta}=\frac{\sin 2\theta}{\sin\theta\cos\theta}$$

$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}=2$$

h) $$\frac{\sec^2\theta}{2-\sec^2\theta}=\frac{\frac{1}{\cos^2\theta}}{2-\frac{1}{\cos^2\theta}}=\frac{\frac{1}{\cos^2\theta}}{\frac{2\cos^2\theta-1}{\cos^2\theta}}=\frac{1}{\cos^2\theta}\cdot\frac{\cos^2\theta}{2\cos^2\theta-1}=\frac{1}{2\cos^2\theta-1}$$

double angle formula $=\cos(2\theta)$

$$=\frac{1}{\cos(2\theta)}=\sec 2\theta$$

---

(29) $v=\langle 3,-5\rangle$, $w=\langle -2,3\rangle$ a) $2v+3w=\langle 6,-10\rangle+\langle -6,9\rangle=\langle 0,-1\rangle$

$2v+3w=-j$

b) $v-w=\langle 3-(-2),-5-3\rangle=\langle 5,-8\rangle$

$\|v-w\|=\sqrt{25+64}=\sqrt{89}$

c) $\|v+w\|=\|\langle 1,-2\rangle\|=\sqrt{1+4}=\sqrt{5}$

d) $\|v\|=\sqrt{9+25}=\sqrt{34}$ $\|w\|=\sqrt{4+9}=\sqrt{13}$, $\|v\|-\|w\|=\sqrt{34}-\sqrt{13}$

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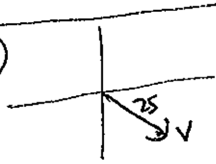
(30) 

$v=(\|v\|\cos 330^\circ)\vec{i}+(\|v\|\sin 330^\circ)\vec{j}=\frac{25\sqrt{3}}{2}i-\frac{25}{2}j$

---

(31) $(r,\theta)=(-4,-\frac{\pi}{4})$ rectangular $x=r\cos\theta=-2\sqrt{2}$, $y=r\sin\theta=2\sqrt{2}$

Ans $(-2\sqrt{2},2\sqrt{2})$

$(r,\theta)=(6,\frac{7\pi}{6})$ rectangular $=(6\cos\frac{7\pi}{6},6\sin\frac{7\pi}{6})=(-3\sqrt{3},-3)$

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(32) $(6,-6)$ $r=\sqrt{36+36}=6\sqrt{2}$, $\tan\theta=-1$ Quad IV, $\theta=-\frac{\pi}{4}$ Ans $(6\sqrt{2},-\frac{\pi}{4})$

$(-3,-3\sqrt{3})$, $r=\sqrt{9+9(3)}=3(2)=6$, $\tan\theta=\sqrt{3}$ Quad III, $\theta=\pi+\frac{\pi}{3}$ Ans $(6,\frac{4\pi}{3})$

Answerkey Review Final Exam MAC1114 10

Multiple choice

M1 D	M2 E	M3 D	M4 B	M5 C	M6 B
M7 E	M8 E	M9 D	M10 A	M11 B	M12 C
M13 D	M14 D	M15 B	M16 B	M17 C	M18 A
M19 A	M20 B	M21 A	M22 C	M23 A	M24 C
M25 D	M26 B	M27 A	M28 D	M29 plor ⟨4, 6⟩	M30 plor ⟨−6, 1⟩
M31 A	M32 B	M33 C	M34 C		