MAC 1140

Review for Final Exam

The exam will consists of problems similar to the ones below. When preparing, focus on understanding and general procedures (when available) rather than specific question. Answers are included

1. Factor completely each polynomial over the real number system.

a) $3x^2 + 14x + 8$
b) $x^4 - 1$
c) $4 - 14x^2 - 8x^4$
d) $x^4 - x^3 + x - 1$
e) $3x^4 - 75$
f) $x^3 - 7x^2 - x + 7$
g) $8x^3 + 125$
h) $7(x^2 - 6x + 9) + 5(x-3)$
i) $3(x-1)^2(x+4)^7 - 5(x-1)^3(x+4)^6$

2. Find the domain of the following functions. Express your answer in interval notation.

a) $f(x) = \dfrac{3x-6}{2x^2+9x+4}$

b) $f(x) = \sqrt{\dfrac{1-x}{x}}$

c) $f(x) = \sqrt[3]{x+2}$

d) $g(x) = \dfrac{\sqrt{x+3}}{x^2-4}$

e) $f(x) = \sqrt{\dfrac{x-1}{x^2-16}}$

3. Find the following values for the function $f(x) = \dfrac{x}{x^2+1}$. Simplify your answer

a) f(0) ; b) f(-x) ; c) f(x + h) ; d) f(2x) ; e) f(-3 + h) ; f) f(2 + h) – f(2)

4. Find the difference quotient $\dfrac{f(x+h) - f(x)}{h}$ of:

a) $f(x) = \dfrac{-3}{x-4}$

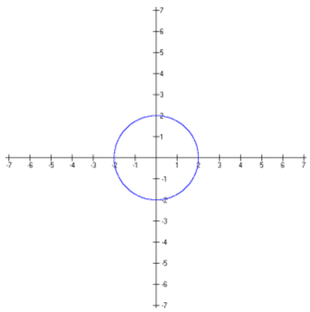
b) $f(x) = -2x^2 + x - 8$

5. Given two functions $f(x) = \dfrac{2}{x}$ and $g(x) = \sqrt{x+1}$. Find the following functions and their domains

a) $f + g$ b) $f - g$ c) $f \cdot g$ d) $\dfrac{f}{g}$

6. Which of the following graphs represents a function. Explain.

A)



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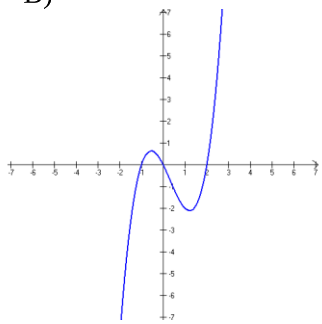

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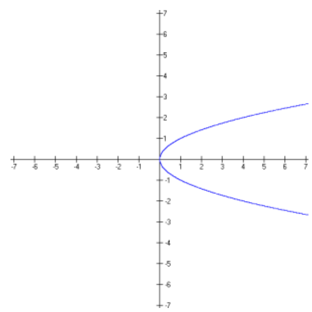

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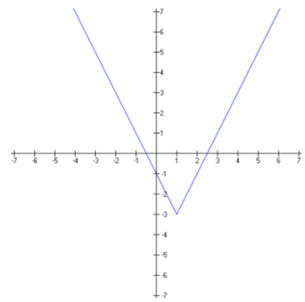
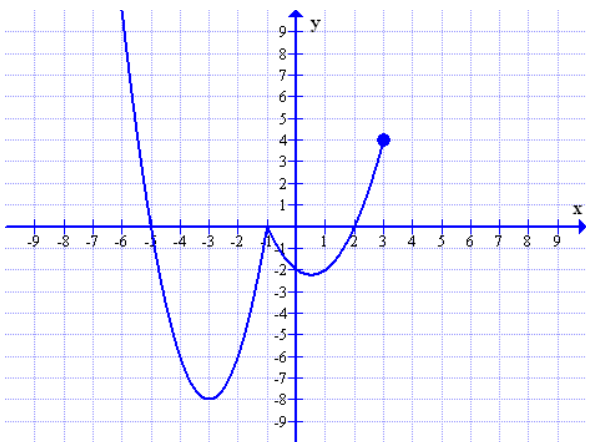


7. Use the graph of the function f given below to answer parts a) - l)



a) Find f(0) and f(-4).
b) Is f(-2) positive or negative?
c) What is the domain of f?
d) What is the range of f?
e) What are the x-intercepts?
f) What is the y-intercept?
g) Find all values of x for which f(x) = -6.
h) List the interval(s) on which f is increasing.
i) List the interval(s) on which f is decreasing.
j) List the interval(s) on which f(x) >0
k) List the interval(s) on which f(x) < 0.
l) Determine whether f is even, odd or neither

8. Determine algebraically whether the function $g(x) = \dfrac{x}{x^2 - 1}$ is even, odd, or neither. Be thorough and precise. Use function notation.

9. Using transformation, graph $f(x) = 3 - 2(x + 3)^5$.. Use a sequence of graphs. Plot accurately at least 3 points for the basic function and use them to perform transformations.

10. Graph piecewise functions

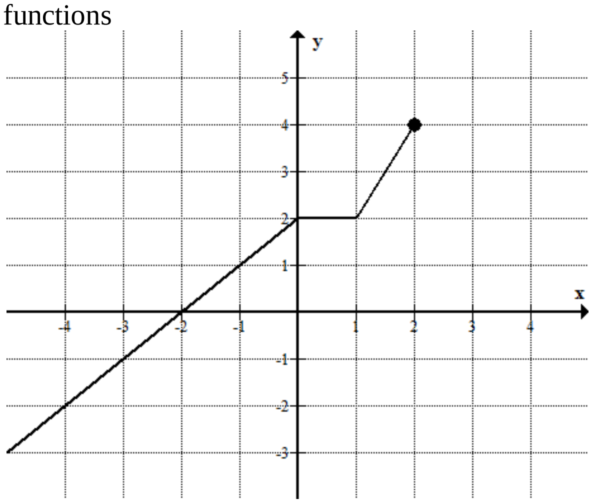
a) $f(x) = \begin{cases} x^3 \text{ if } x < 1 \\ -2 \text{ if } x = 1 \\ \sqrt{x} \text{ if } x > 1 \end{cases}$

b)

$$f(x)=\begin{cases} 2x-1 & if\ x\le -2 \\ 4 & if -2<x<-1 \\ -x^2+4 & if\ x>-1 \end{cases}$$

c) $$f(x)=\begin{cases} \frac{1}{x} & if\ \ x<-1 \\ |x| & if\ \ -1\le x\le 1 \\ -2 & if\ \ x>1 \end{cases}$$

11. The graph of a function f is given below. Use the graph of f as the first step toward graphing each of the following functions



a) h(x) = f(x-1) +3
b) g(x) = f(-x)
c) p(x) = $\frac{1}{2}f(x)-2$
d) r(t) = f(2x)

12. Given the functions $f(x)=\frac{x}{x+3}$ and $g(x)=\frac{5}{x}$, find and simplify $f\circ g$ and $g\circ f$. Determine their domains.

13**.** The function f is one-to-one. Find the inverse function and check your answer. Determine the domain and range for the function and its inverse.

a) $f(x)=\frac{2x+3}{x+2}$

b) $f(x)=3-\sqrt[3]{x-5}$
c) $f(x)=2\log_3(x-1)$

14. Use synthetic division to divide $4x^4 - 3x^3 - x^2 + 5$ by x + 2.

15. Find the remainder when $2x^5 + x^3 - 4x^2 + 2$ is divided by x-2.

16. Show that $(x-3)^2$ is a factor of the polynomial $p(x) = x^4 -3x^3-19x^2 + 87x - 90$.

17 . Function $p(x) = 2x^3 + x^2 - 2x - 1$ is given

a) List real zeros and their multiplicities

b) Determine whether the graph crosses or touches the x-axis at the x-intercept.

c) Determine the end behavior.

d) Sketch the graph of p(x)

18. Let $p(x) = -2x^3(x-3)(x+4)^2 (x^2 +5)$

a) List real zeros and their multiplicity

b) Determine whether the graph crosses or touches the x-axis at each x-intercept.

c) Determine the end behavior of p.

d) Sketch the graph of p(x)

19. Sketch the graph of the given rational function by finding the domain, x-intercept(s), y-intercept, asymptotes (vertical, horizontal oblique), checking for symmetry, finding the points where the graph crosses the horizontal/oblique asymptote and constructing the sign chart to determine where the graph is above/below the x-axis.

a) $f(x) = \dfrac{2x^2 - 4x}{x^2 - 3x - 4}$

a) $f(x) = \dfrac{x^2 - x - 12}{x+1}$.

20. Find the vertical, horizontal or oblique asymptotes, if any, of the rational function

a) $\mathrm{f(x)} = \dfrac{2x^2 - 5x + 2}{x^2 - 4}$

b) $\mathrm{f(x)} = \dfrac{x^3 + 3x^2 + 5}{x^2 + 3}$

c) $f(x) = \dfrac{x^2 - 1}{x^3 + 2x^2}$

21. Solve the inequality; write the answer in the interval notation

a) $x^3 + 4x^2 \geq x + 4$

b) $\dfrac{-2}{1-3x} < 1$

c) $\dfrac{x(x^2 + x - 2)}{x^2 + 9x + 20} \leq 0$

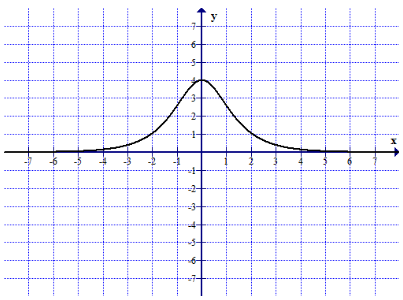
d) $\dfrac{5}{x+2} \geq \dfrac{3}{x+1}$

22. Let $f(x) = |x-2|$ and $g(x) = \dfrac{2}{x+1}$. Find a) $(f \circ g)(4)$ b) $(g \circ f)(2)$ c) $(f \circ f)(1)$ d) $(g \circ g)(0)$

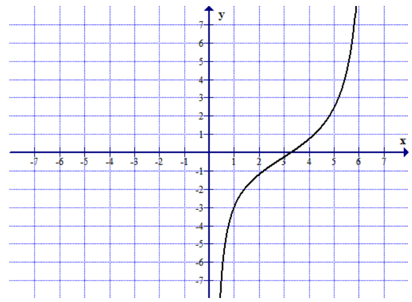
23. Find functions *f* and g so that $f \circ g = H$, where $H(x) = \sqrt{x^2 + 3x - 2}$.

24.Determine whether the following functions are one-to-one. Explain

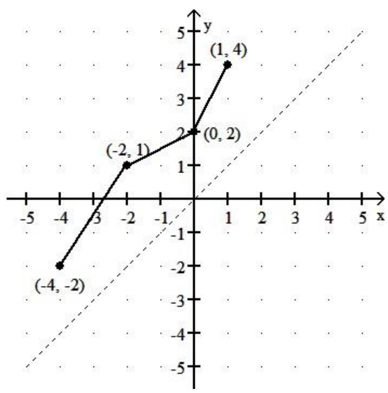
a)



b)



25. Use the graph of the given one-to-one function to sketch the graph of the inverse function.



26. Change an exponential expression to an equivalent expression involving a logarithm

a) $2.2^N = 5$

b) $e^x = 8$

27. Change each logarithmic expression to an equivalent expression involving an exponent

a) $\log_b 4 = 2$

b) lnx = 4

28. Find the exact value of each logarithm without using a calculator

a) $\log_{1/2} 4$ b) $\log_3 \frac{1}{27}$ c) $\ln e^3$ d) $\log_5 5^{4.2}$

29. Use transformation to graph $f(x) = -3^{-x+1} + 2$. Plot accurately at least 3 points on the graph of a basic function and use them to perform the transformations. Determine domain, range and the equation of horizontal asymptote

30**.** Find the domain of the given functions

a) $f(x) = \log_7\left(\frac{x+1}{x}\right)$

b) $f(x) = x + \log_4\left(\frac{x+1}{9-x^2}\right)$

31. Given f(x) = - ln(2x+4)

a) Find the domain of f

b) Use transformations to graph f

c) Use the graph to find the range and any asymptotes of f

d) Find the inverse function f^{-1}

e) Graph the inverse function

32. Let $f(x)=2+\log_3(x+1)$.

a) Use transformations to graph function f.

b) Use the graph to find the range of f and any asymptotes

33. (a) Rewrite the expression as a single logarithm and simplify

$$8\log_2\sqrt{3x-2}-\log_2\left(\frac{4}{x}\right)+\log_2 4$$

(b) Express as a single logarithm. Assume all variables are positive and b ≠ 1

$$2\log_b t-\frac{4}{5}\log_b s+\frac{1}{3}\log_b v-4\log_b u$$

34. Rewrite the expression as a sum/difference of logarithms. Express powers as factors.

a) $\ln\left(\dfrac{7x\sqrt[3]{1+6x}}{(x+1)(x-3)^2}\right)$; assume x >3

b) $\ln\left(\dfrac{(x-2)^2}{x^4\sqrt{x-3}}\right)$; assume x >3

35. Use properties of logarithms to find the exact value of each expression

a) $\log_6 9+\log_6 4$

b) $2^{\log_2 5}$

c) $\log_2 6\cdot\log_6 4$

36. Solve the equations

a) $2^{x+1}=5^{1-2x}$

b) $\log_3 x+\log_3(x-2)=\log_3(x+4)$

c) ln(x+1)- ln x = 2

d) $5(2^{3x})=8$

e) $3^{2x}+3^x-2=0$

f) $4^{x^2+4x}=\dfrac{1}{64}$

g) $\log_2(x+4)+\log_2(x+3)=1$

h) $5e^x-2=23$

i) $\log_3(x^2+x+7)=2$

j) $\left(e^4\right)^x e^{x^2}=e^{12}$

k) $\log_x 81=4$

l) $e^{2x+5}=\dfrac{1}{3}$

37. Use the Change-of-the base formula and a calculator to approximate $\log_7 5$.

38.Find the standard equation of the parabola with given properties. Graph the equation.

a) Vertex at (0, 0); axis of symmetry the x-axis; containing the point (4, 9)

b) Focus at (-2, 0); directrix the line x = 2

c) Focus at (-3,-2); directrix the line y = 2

39. Find the vertex, focus, directrix, the coordinates of the endpoints of the latus rectum segment of the parabola given by the equation $y^2 + 12y = -2x - 30$. Graph the parabola.

40. Find the equation of the ellipse with given properties. For each ellipse find the center, foci, and vertices. Find the lengths of major and minor axes. Graph the equation.
a) Focus at (-6,0); vertices (±7, 0).
b) Foci at (1,2) and (-3,2); vertex at (- 4,2)
c) Center at (2,-1); vertex at (7,-1); focus at (4,-1)

41. Find the center, foci, and vertices of the ellipse $4x^2 + 9y^2 + 8x - 18y = 23$. Graph the ellipse.

42. Find the equation of the hyperbola with given properties. For each hyperbola find the center, vertices, foci, and the equations of the asymptotes. Graph the equation

a) vertices at (±4,0); focus at (5,0)
b) Foci at (0,±2); asymptote the line y = -x
c) Focus at (-4,0); vertices at (-4,4) and (-4,2)

43. Graph the conic
a) $4x^2 + 25y^2 + 24x - 50y - 39 = 0$
b) $4y^2 - 9x^2 + 24y + 36x + 36 = 0$

44. Identify the graph of a given equation as a circle, ellipse, parabola, hyperbola, not a conic.
a) $2x + 3y = 6$
b) $2x^2 - 5y^2 + 7x - 6y - 4 = 0$
c) $4y^2 + 2x - 4y + 1 = 0$
d) $3x^2 + 7y^2 - 2x + 3y - 2 = 0$
e) $x^2 + y^2 - 5x - 11y + 2 = 0$

45. Solve the following system of equations. If there are no solutions, say so. If there are infinitely many solutions, describe the solution set.

a) $\begin{cases} 5x - y = 13 \\ 2x + 3y = 12 \end{cases}$

b) $\begin{cases} 2x + y = 1 \\ 4x + 2y = 3 \end{cases}$

c) $\begin{cases} x + 2y = 4 \\ 2x = 8 - 4y \end{cases}$

d) $\begin{cases} xy = 4 \\ 2x^2 - xy + y^2 = 8 \end{cases}$

46. Solve the linear system by elimination

$$\begin{cases} x - y - z = 1 \\ -x + 2y - 3z = -4 \\ 3x - 2y - 7z = 0 \end{cases}$$

47. Evaluate the determinants

$$\begin{vmatrix} -3 & -1 \\ 4 & 2 \end{vmatrix}$$

48. Solve the system using Cramer's Rule. If Cramer's Rule is not applicable, say so.

a) $\begin{cases} 3x - 6y = 24 \\ 5x + 4y = 12 \end{cases}$

b) $\begin{cases} 3x - 2y = 4 \\ 6x - 4y = 0 \end{cases}$

49. Graph each equation of the system and solve it to find points of intersection

a) $\begin{cases} xy = 3 \\ x^2 + y^2 = 10 \end{cases}$

b) $\begin{cases} 2y - x = 2 \\ x^2 - 4y^2 = 16 \end{cases}$

50. Write down the first five terms of the sequence

a) $\{a_n\} = \left\{ \frac{3^n}{n} \right\}$

b) $\{d_n\} = \left\{ (-1)^n \left(\frac{n}{2n-1} \right) \right\}$

c) $a_1 = -1, \quad a_n = n + 3a_{n-1}$

51. Write the formula for the n^{th} term of a sequence $\{a_n\}$given by

a) -1, 2,-3, 4, -5, 6, …

b) $\frac{1}{1 \cdot 2}, \frac{1}{2 \cdot 3}, \frac{1}{3 \cdot 4}, \frac{1}{4 \cdot 5}, \cdots$

52. Write out each sum

a) $\sum_{k=0}^{n} \frac{1}{3^k}$

b) $\sum_{k=1}^{n} \frac{k^2}{2}$

c) $\sum_{j=5}^{10} (-1)^j (2j + 3)$

53. Express each sum using summation notation. Use 1 as the lower limit of summation and i for the index of summation.

a) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \cdots + \frac{13}{14}$

a) $3^2 + 6^3 + 9^4 + \ldots + 24^9 + \ldots$

b) $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \cdots + \frac{1}{729}$

54. Find the sum

a) $\sum_{k=1}^{40} k$

b) $\sum_{k=1}^{52} (3k - 7)$

c) $\sum_{k=1}^{50} (2k + 1)^2$

55. Show that the sequence $\{a_n\} = \{3n + 7\}$, $n \geq 1$ is arithmetic. Find the first term and the common difference

56. Find the 80th term of the arithmetic sequence $1, -2, -5, \ldots$

57. Find the nth-term of the arithmetic sequence if
a) $a_1 = -2, \quad d = 4$; d is the common difference
b) 5th term is -2 and the 13th term is 30 (You must use a system of equations to solve this problem).

58. Consider the finite arithmetic sequence: 73, 78, 83, 88, … , 558

a) Write the nth term by finding the first term and the common difference.
b) Write the sum of all its terms in sigma form.
c) Find the value of the sum by using an appropriate formula.

59. Find the sum of the first 38 terms of the arithmetic sequence 3, 8, 13, 18, 23, …

60. Find x so that x + 3, 2x + 1 and 5x + 2 are consecutive terms of an arithmetic sequence.

61. Show that the sequence $\{a_n\} = \left\{ \dfrac{2^n}{3^{n-1}} \right\}$ is geometric. Find the first term and the common ratio.

62. Find the fifth and the n-th term of the geometric sequence for which $a_1 = -2, \quad r = -1/3$

63. Find the 10-th term of the geometric sequence -1, 2, -4, ….

64. Find the common ratio and the formula for the n-th term of the geometric sequence given
a) $a_{19} = 786432, \quad r = -2$
b) $a_5 = 32/81, \quad a_8 = 256/2187$

65. Consider the geometric sequence: $2, \dfrac{6}{5}, \dfrac{18}{25}, \ldots, 2\left(\dfrac{3}{5}\right)^{n-1}, \cdots$

a) Write the formula for the n-th term.
b) Write the sum of the first n terms of this sequence using sigma notation.
c) Find the formula for the sum of the first n terms of this sequence.
d) Find the sum of the first 10 terms of this sequence

66. Evaluate without using a calculator
a) 5!
b) $\dbinom{12}{3}$

67. Use the Binomial Theorem to expand given binomial
a) $(3x + 2)^7$
b) $(x - 2y)^5$

68. Find the coefficient of x^4 in the expansion of
a) $(4x + 1)^7$
b) $\left(x - \dfrac{2}{\sqrt{x}} \right)^{10}$

69. Find the 7-th term of $(2x + 3)^9$.

ANSWERS:

1)

a) $(3x+2)(x+4)$; b) $(x^2+1)(x+1)(x-1)$; c) $-2(2x+1)(2x-1)(x^2+2)$

d) $(x+1)(x-1)(x^2-x+1)$; e) $3(x+\sqrt{5})(x-\sqrt{5})(x^2+5)$; f) $(x-7)(x+1)(x-1)$

g) $(2x+5)(4x^2-10x+25)$; h) $(7x-16)(x-3)$; i) $(x-1)^2(x+4)^6(-2x+17)$

2)

a) $(-\infty,-4)\cup(-4,-\frac{1}{2})\cup(-\frac{1}{2},+\infty)$

b) $(0,1]$

c) $(-\infty,+\infty)$

d) $[-3,-2)\cup(-2,2)\cup(2,+\infty)$

e) $(-4,1]\cup(4,+\infty)$

3)

a) 0; b) $\dfrac{-x}{x^2+1}$; c) $\dfrac{x+h}{(x+h)^2+1}$; d) $\dfrac{2x}{4x^2+1}$; e) $\dfrac{h-3}{h^2-6h+10}$; f) $-\dfrac{2h^2+3h}{5(h^2+4h+5)}$

4) a) $\dfrac{3}{(x+h-4)(x-4)}$; b) $-4x-2h+1$

5) a) $\dfrac{2}{x}+\sqrt{x+1}$; $[-1,0)\cup(0,+\infty)$

b) $\dfrac{2}{x}-\sqrt{x+1}$; $[-1,0)\cup(0,+\infty)$

c) $\dfrac{2\sqrt{x+1}}{x}$; $[-1,0)\cup(0,+\infty)$

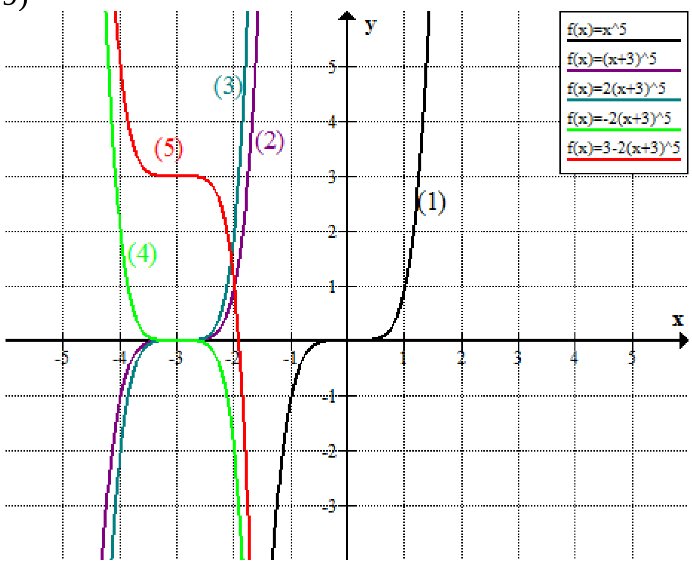
d) $\dfrac{2}{x\sqrt{x+1}}$; $(-1,0)\cup(0,+\infty)$

6) B and D are functions; pass the Vertical Line Test

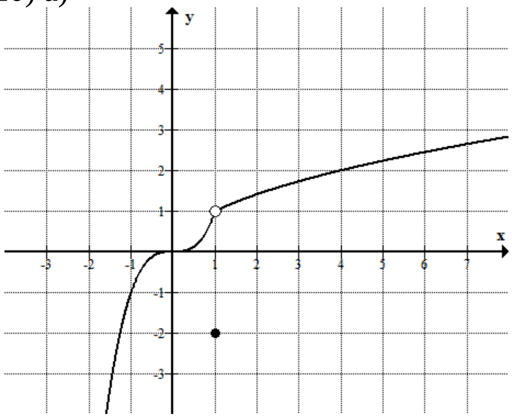
7) a) f(0)= -2; f(-4) = -6 ; b) negative; c) (-∞, 3]; d) [-8, +∞); e) (-5,0), (-1,0), (2,0); f) (0, -2); g) -4, -2; h) (-3,-1)∪(½, 3) ;i) (-∞, -3)∪(-1, ½) ; j) (-∞, -5)∪(2,3] ; k) (-5,-1)∪(-1,2) ; l) neither

8) odd

9)



10) a)



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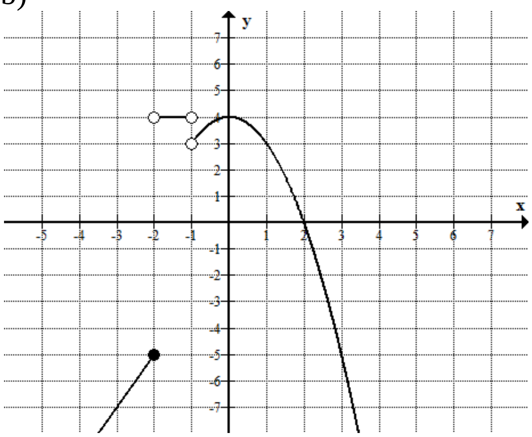

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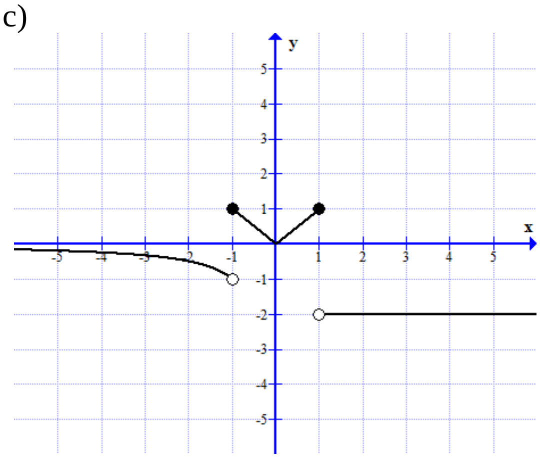
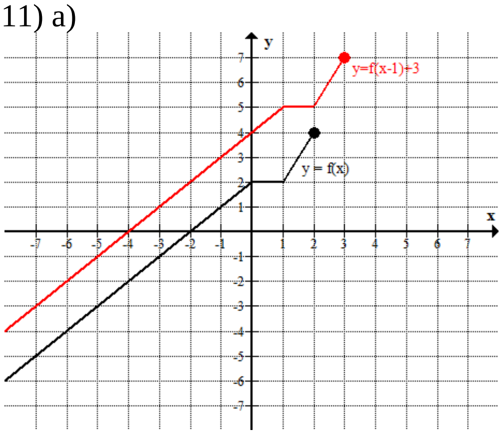


11) a)



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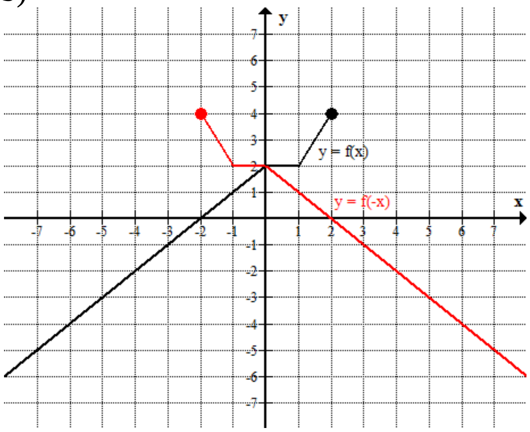

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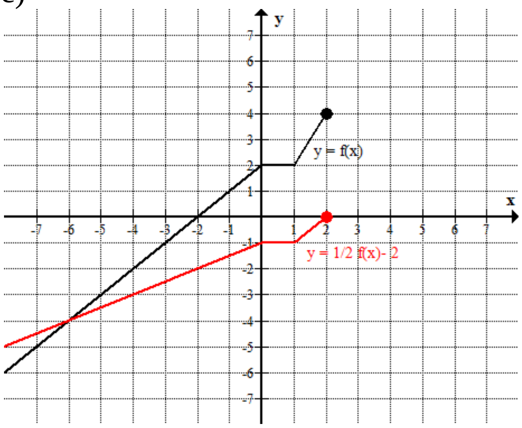

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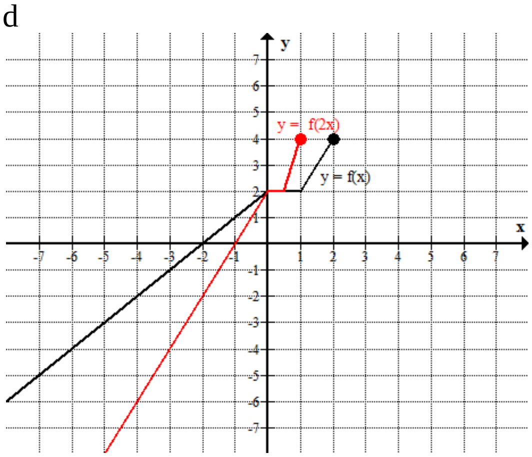


12) $(f\circ g)(x)=\dfrac{5}{3x+5}$, domain= {x| x≠0, -5/3}; $(g\circ f)(x)=\dfrac{5(x+3)}{x}$, domain= {x| x≠ -3, 0}

13)a) $f^{-1}(x)=\dfrac{3-2x}{x-2}$, domain of f = {x| x≠ -2}; range of f = {y| y≠ 2}; domain of f^{-1} = {x| x≠ 2};
range of f^{-1} = {y| y ≠ -2};
b) $f^{-1}(x)=(3-x)^3+5$, domain of f = (-∞,+∞)range of f = (-∞,+∞)domain of f^{-1} = (-∞,+∞)
range of f^{-1} = (-∞,+∞)
c) $f^{-1}(x)=3^{x/2}+1$, domain of f = (1,+∞) range of f = (-∞,+∞)domain of f^{-1} = (-∞,+∞)
range of f^{-1} = (1,+∞)

14) Quotient: $4x^3 - 11x^2 + 21x - 42$; remainder: 89

15) 58

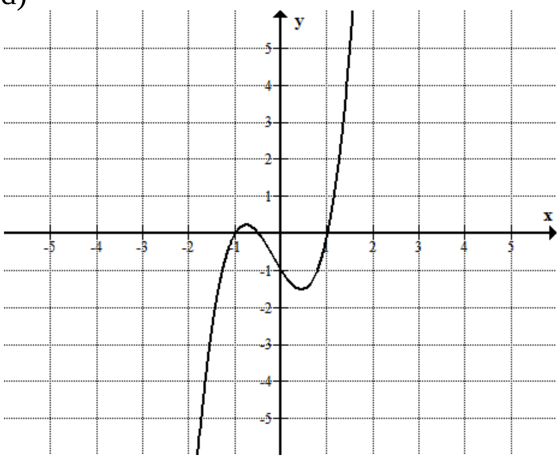
16) p(x) = $(x-3)^2 (x+5)(x-2)$

17)a) -1, multiplicity 1; 1, multiplicity 1; -1/2, multiplicity 1

b) graph crosses the x-axis at each x-intercept

c) $y = 2x^3$

d)



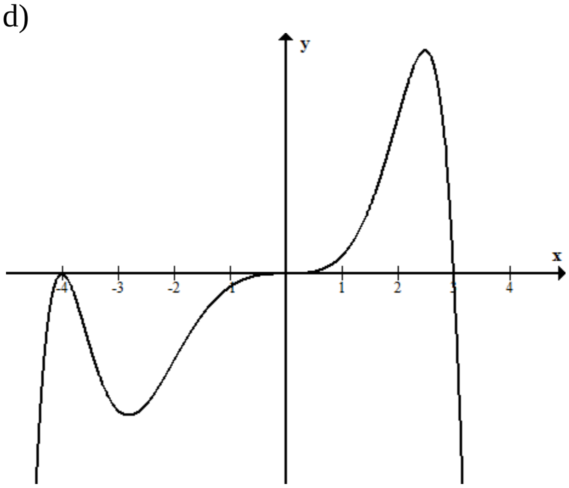
18)

a) 0, multiplicity 3; 3, multiplicity 1; -4, multiplicity 2

b) crosses as cube function at 0; crosses as a straight line at 3; touches at -4

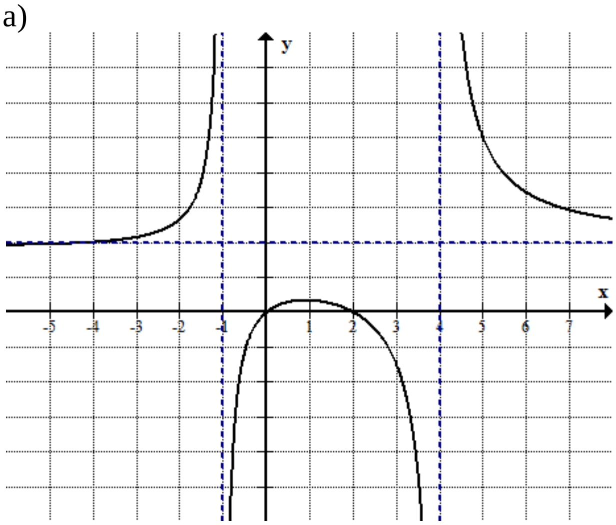
c) $y = -2x^8$

d)

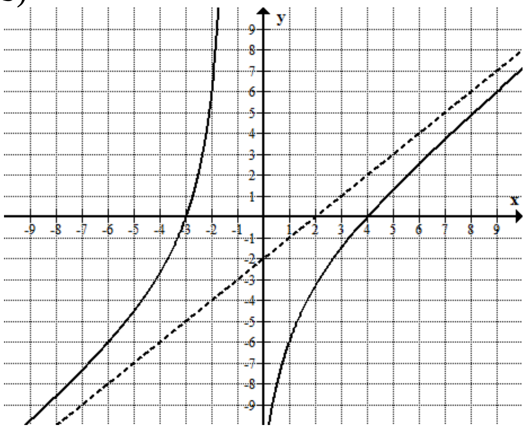


19)

a)



b)



20) a) Horizontal asymptote: y = 2; vertical asymptote: x= -2

b) No vertical or horizontal asymptotes; oblique asymptote: y = x+3

c) Horizontal asymptote: y = 0; vertical asymptotes" x =0, x = -2

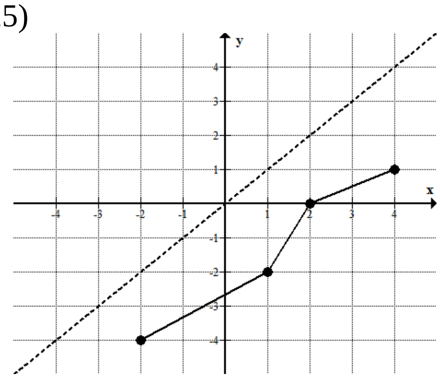
21) a) $[-4,-1]\cup[1,+\infty)$; b) (1/3, 1) ; c) $(-\infty, -5)\cup(-4,-2]\cup[0,1]$; d) $(-2,-1)\cup[\frac{1}{2}, +\infty)$

22) a) 8/5; b) 2 ; c) 1 ; d) 2/3

23) $f(x)=\sqrt{x};\ g(x)=x^2+3x-2$

24) a)No; fails the Horizontal Line Test; b) Yes; passes the Horizontal Line Test

25)

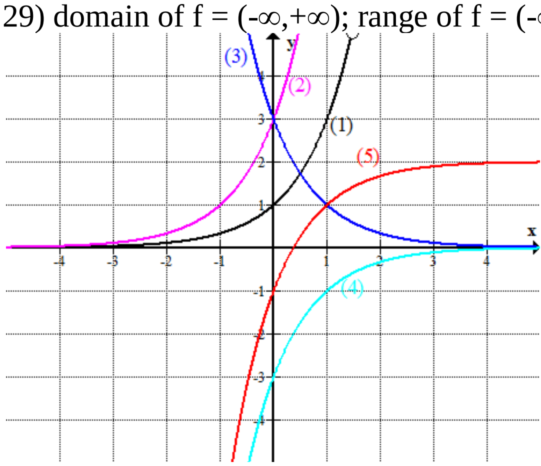


26)a) $N=\log_{2.2}(5)$; b) x = ln(8)

27) a) $b^2 = 4$; b) $e^4 = x$

28) a) -2 ; b) -3 ; c) 3 ; d) 4.2

29) domain of f = $(-\infty,+\infty)$; range of f = $(-\infty,2)$; horizontal asymptote: y = 2

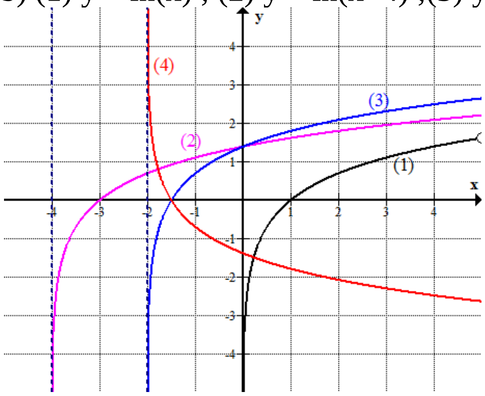


(1) $y = 3^x$; (2) $y = 3^{x+1}$; (3) $y = 3^{-x+1}$; (4) $y = -3^{-x+1}$; (5) $y = -3^{-x+1}+2$

30) a) $(-\infty,-1)\cup(0,+\infty)$; b) $(-\infty,-3)\cup(-1,3)$

31) a) domain of f = $(-2, +\infty)$

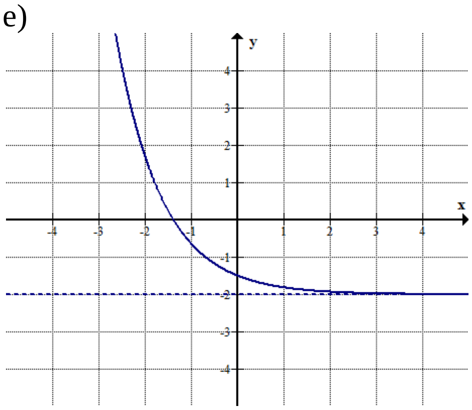
b) (1) y = ln(x) ; (2) y = ln(x+4) ;(3) y = ln(2x+4) ; (4) y = -ln(2x+4)



c) Range of f = (-∞,+∞) ; vertical asymptote: x = -2

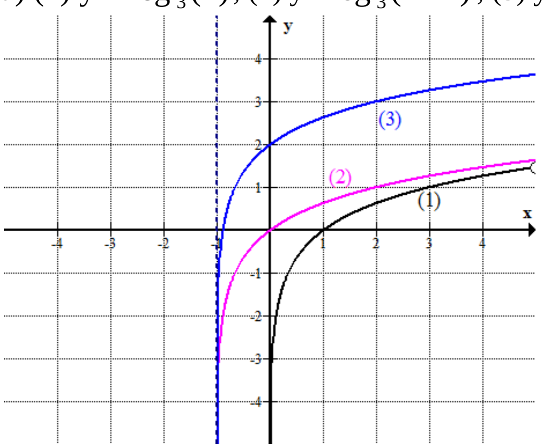
d) $f^{-1}(x) = \frac{1}{2}e^{-x} - 2$

e)



32)

a) (1) y = $\log_3(x)$; (2) y = $\log_3(x+1)$; (3) y = $2 + \log_3(x+1)$



b) range of f = (-∞,+∞) ; vertical asymptote: x = -1

33) a) $\log_2\left(x(3x-2)^4\right)$; b) $\log_2\left(\frac{t^2 \cdot \sqrt[3]{v}}{u^4 \cdot \sqrt[5]{s^4}}\right)$

34) a) $\ln(7) + \ln(x) + \frac{1}{3}\ln(1+6x) - \ln(x+1) - 2\ln(x-3)$

b) 2ln(x-2) – 4lnx – ½ ln(x-3)

35) a) 2 ; b) 5; c) 2

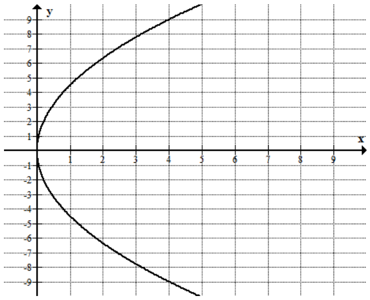
36) a) $\frac{\ln(5/2)}{\ln(50)}$; b) 4 ; c) $\frac{1}{e^2-1}$; d) $\frac{\ln(8/5)}{3\ln(2)}$; e) 0; f) -1, -3; g) -2 ; h) ln(5) ; i) -2, 1; j) -6, 2; k) 3; l) $\frac{-5-\ln(3)}{2}$

37) 0.827

38)

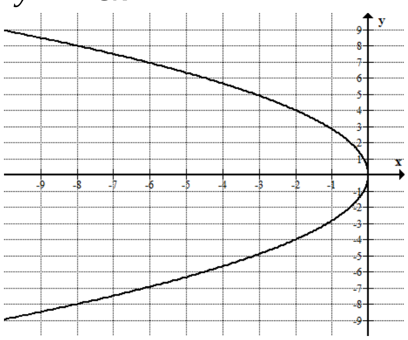
a)

$$y^2 = \frac{81}{4}x$$

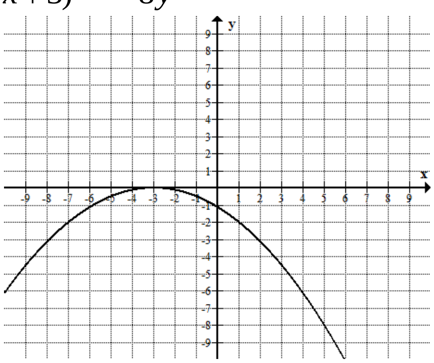


b)

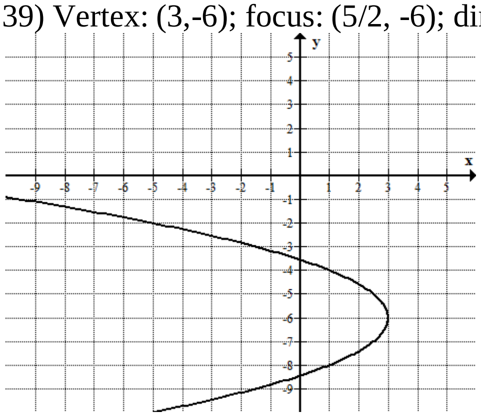
$$y^2 = -8x$$



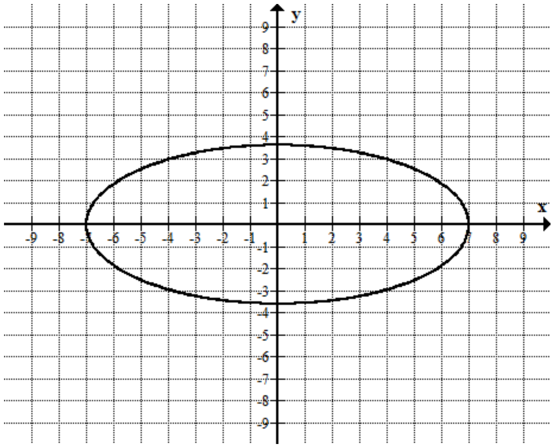
c) $(x+3)^2 = -8y$



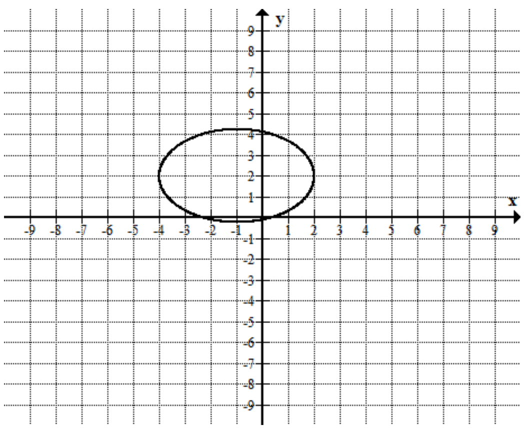
39) Vertex: (3,-6); focus: (5/2, -6); directrix: x = 7/2 ; latus rectum endpoints: (5/2, -5) (5/2, -7)



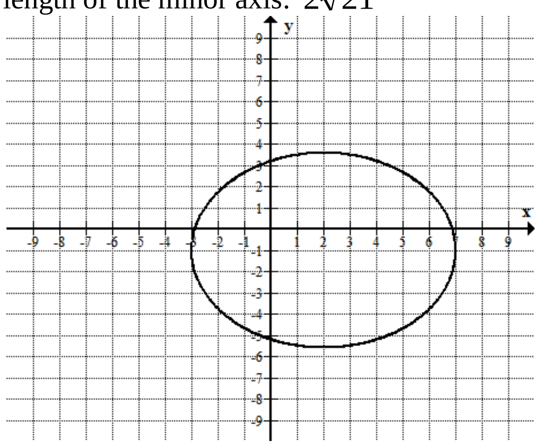
40) a) $\frac{x^2}{49}+\frac{y^2}{13}=1$; Center: (0,0); Foci: (±6, 0) ; Vertices: (±7, 0); length of the major axis: 14; length of the minor axis: $2\sqrt{13}$



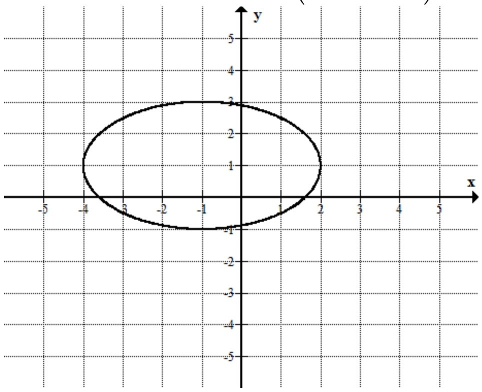
b) $\frac{(x+1)^2}{9}+\frac{(y-2)^2}{5}=1$; Center: (-1,2); Foci: (1, 2), (-3,2) ; Vertices: (-4, 2), (2,2); length of the major axis: 6; length of the minor axis: $2\sqrt{5}$



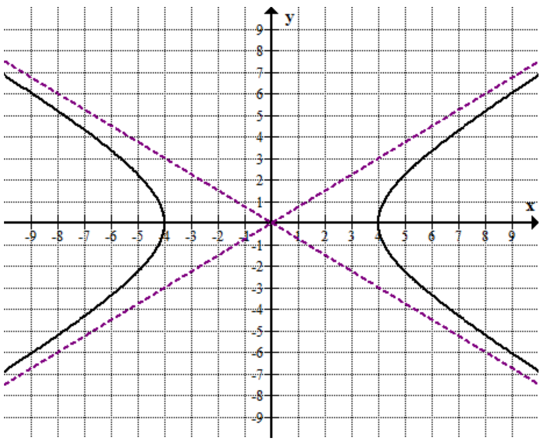
c) $\frac{(x-2)^2}{25}+\frac{(y+1)^2}{21}=1$; Center: (2,-1); Foci: (4,-1), (0,-1) ; Vertices: (7, -1), (-3,-1); length of the major axis: 10; length of the minor axis: $2\sqrt{21}$



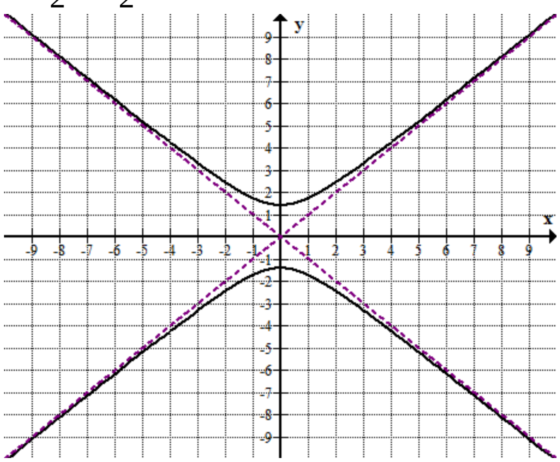
41) Center: (-1,1); Foci: $\left(-1 \pm \sqrt{5}, 1\right)$; Vertices: (2, 1), (-4,1);



42)a) $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$; Center: (0,0); Foci: (± 5,0) ; Vertices: (± 4, 0); asymptotes: $y = \pm \dfrac{3}{4} x$

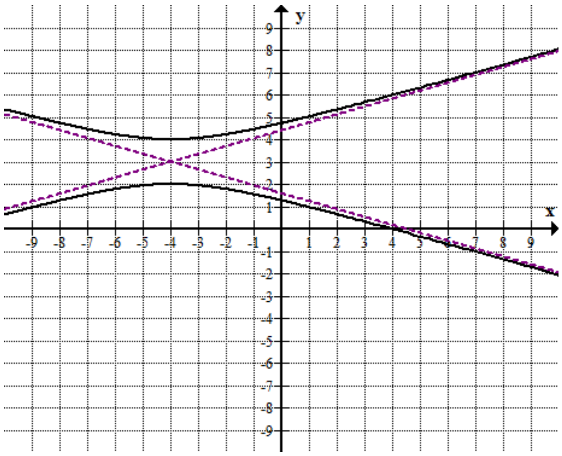


b) $\dfrac{y^2}{2} - \dfrac{x^2}{2} = 1$; Center: (0,0); Foci: (0,± 2) ; Vertices: $\left(0, \pm\sqrt{2}\right)$asymptotes: $y = \pm x$



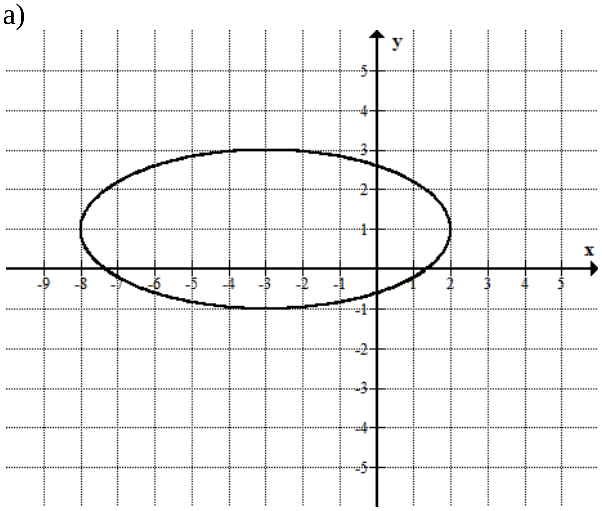
c) $\dfrac{(y-3)^2}{1} - \dfrac{(x+4)^2}{8} = 1$; Center: (-4,3); Foci: (-4,0), (-4,6) ; Vertices: (-4,4), (-4,2); asymptotes:

$y = \pm \dfrac{\sqrt{2}}{4}(x+4) + 3$

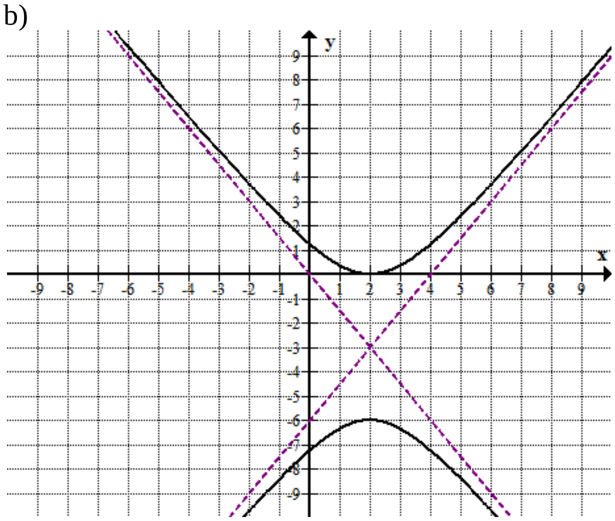


43)

a)

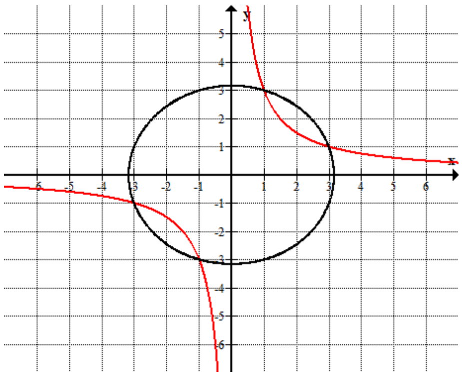


b)

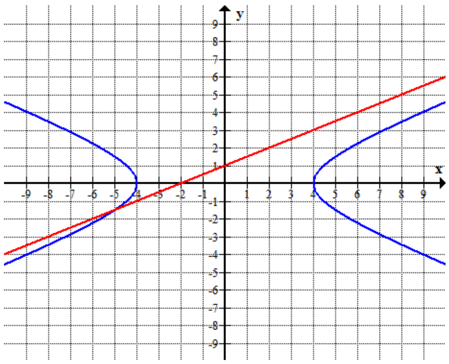


44) a) not a conic; b) hyperbola ; c) parabola ; d) ellipse ; e) circle
45) a) (3,2) ; b) no solution ; c) infinitely many solutions, solution set = { (x,y)| x = 4-2y, y is any real number}
46) infinitely many solutions; solutions set = {(x,y,z)| x = 5z -2, y = 4z-3, z is any real number}
47) -2 ;
48) a) (4,-2) ; b) not applicable;

49) a) solutions: (1, 3), (3, 1), (-1,-3), and (-3, -1)



b) Solution (-5, -3/2)



5o) a) $3, \frac{9}{2}, \frac{27}{3}, \frac{81}{4}, \frac{243}{5}$; b) $-1, \frac{2}{3}, -\frac{3}{5}, \frac{4}{7}, -\frac{5}{9}$; c) -1, -1, 0, 4, 17

51) a) $a_n = (-1)^n n, \quad n \geq 1$; b) $a_n = \frac{1}{n(n+1)}, \quad n \geq 1$

52) a) $\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \cdots + \frac{1}{3^n}$; b) $\frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \cdots + \frac{n^2}{2}$; c) -13 + 15 – 17 +19 – 21 + 23

53) a) $\sum_{i=1}^{13} \frac{i}{i+1}$; b) $\sum_{i=1}^{\infty} (3i)^{i+1}$; c) $\sum_{i=1}^{6} (-1)^{i+1} \frac{1}{3^{i-1}}$

54) a) 820 ; b) 3770 ; c) 176,850

55) $a_{n+1} - a_n = 3; \quad a_1 = 10, \quad d = 3$

56) -236

57) a) 4n-6 ; b) 4n- 22

58) a) 5n + 68 ; b) $\sum_{i=1}^{98} (5i + 68)$; c) 30,919

59) 3629

60) x = -3/2

61) $\frac{a_{n+1}}{a_n} = \frac{2}{3}; \quad a_1 = 2, \quad r = \frac{2}{3}$

62) $a_5 = -\frac{2}{81}$; $a_n = -2\left(-\frac{1}{3}\right)^{n-1}$

63) 512

64) a) $r = -2$; $a_n = 3(-2)^{n-1}$;b) $r = \frac{2}{3}$; $a_n = 2\left(\frac{2}{3}\right)^{n-1}$

65) a) $a_n = 2\left(\frac{3}{5}\right)^{n-1}$; b) $\sum_{i=1}^{n} 2\left(\frac{3}{5}\right)^{i-1}$; c) $5\left(1-\left(\frac{3}{5}\right)^{n}\right)$; d) $5\left(1-\left(\frac{3}{5}\right)^{10}\right)$

66) a) 120 ; b) 220

67) a) $2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128$

b) $x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$

68) a) 8960 ; b) 3360

69) $489{,}888x^3$