

**MAC 1140****Review for Final Exam**

The exam will consist of problems similar to the ones below. When preparing, focus on understanding and general procedures (when available) rather than specific questions.

Answers for the problems can be found at the end of this packet starting on Page 12.

1. Factor completely each polynomial over the real number system.

- a)  $3x^2 + 14x + 8$
- b)  $x^4 - 1$
- c)  $4 - 14x^2 - 8x^4$
- d)  $x^4 - x^3 + x - 1$
- e)  $3x^4 - 75$
- f)  $x^3 - 7x^2 - x + 7$
- g)  $8x^3 + 125$
- h)  $7(x^2 - 6x + 9) + 5(x - 3)$
- i)  $3(x - 1)^2(x + 4)^7 - 5(x - 1)^3(x + 4)^6$

2. Find the domain of the following functions. Express your answer in interval notation.

a)  $f(x) = \frac{3x - 6}{2x^2 + 9x + 4}$

b)  $f(x) = \sqrt{\frac{1 - x}{x}}$

c)  $f(x) = \sqrt[3]{x + 2}$

d)  $g(x) = \frac{\sqrt{x + 3}}{x^2 - 4}$

e)  $f(x) = \sqrt{\frac{x - 1}{x^2 - 16}}$

3. Find the following values for the function  $f(x) = \frac{x}{x^2 + 1}$ . Simplify your answer

a)  $f(0)$ ; b)  $f(-x)$ ; c)  $f(x + h)$ ; d)  $f(2x)$ ; e)  $f(-3 + h)$ ; f)  $f(2 + h) - f(2)$

4. Find the difference quotient  $\frac{f(x + h) - f(x)}{h}$  of:

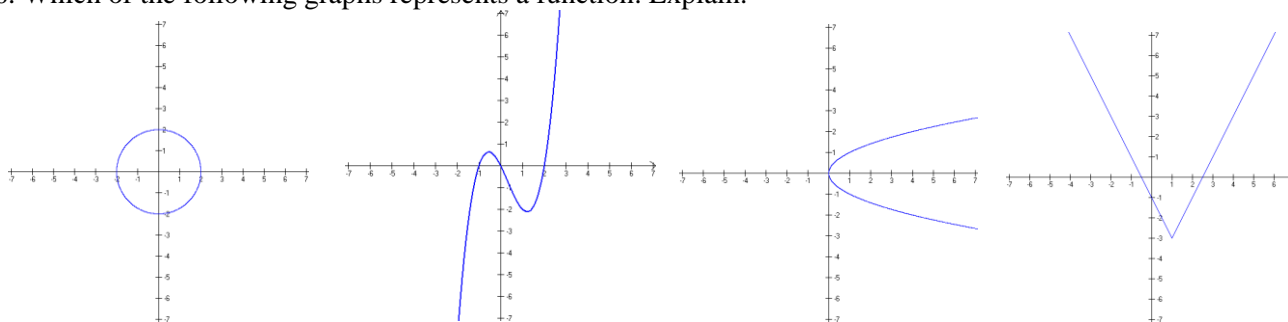
a)  $f(x) = \frac{-3}{x - 4}$

b)  $f(x) = -2x^2 + x - 8$

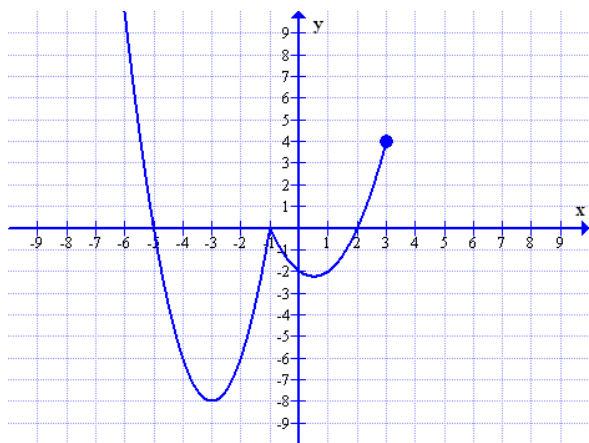
5. Given two functions  $f(x) = \frac{2}{x}$  and  $g(x) = \sqrt{x + 1}$ . Find the following functions and their domains

- a)  $f + g$       b)  $f - g$       c)  $f \cdot g$       d)  $\frac{f}{g}$

6. Which of the following graphs represents a function. Explain.



7. Use the graph of the function  $f$  given below to answer parts a) - l)



- Find  $f(0)$  and  $f(-4)$ .
- Is  $f(-2)$  positive or negative?
- What is the domain of  $f$ ?
- What is the range of  $f$ ?
- What are the  $x$ -intercepts?
- What is the  $y$ -intercept?
- Find all values of  $x$  for which  $f(x) = -6$ .
- List the interval(s) on which  $f$  is increasing.
- List the interval(s) on which  $f$  is decreasing.
- List the interval(s) on which  $f(x) > 0$
- List the interval(s) on which  $f(x) < 0$ .
- Determine whether  $f$  is even, odd or neither

8. Determine algebraically whether the function  $g(x) = \frac{x}{x^2 - 1}$  is even, odd, or neither. Be thorough and precise. Use function notation.

9. Using transformation, graph  $f(x) = 3 - 2(x + 3)^5$ . Use a sequence of graphs. Plot accurately at least 3 points for the basic function and show how they change.

10. Graph the piecewise functions

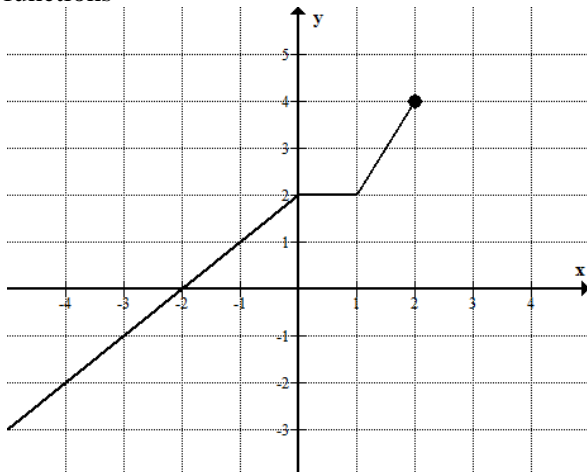
$$a) f(x) = \begin{cases} x^3 & \text{if } x < 1 \\ -2 & \text{if } x = 1 \\ \sqrt{x} & \text{if } x > 1 \end{cases}$$

b)

$$f(x) = \begin{cases} 2x-1 & \text{if } x \leq -2 \\ 4 & \text{if } -2 < x < -1 \\ -x^2 + 4 & \text{if } x > -1 \end{cases}$$

c)  $f(x) = \frac{1}{x}$ , if  $x \leq -2$ ;  $f(x) = 4$ , if  $-2 < x < -1$ ;  $f(x) = \sqrt[3]{x}$ , if  $x > -1$

11. The graph of a function  $f$  is given below. Use the graph of  $f$  as the first step toward graphing each of the following functions



a)  $h(x) = f(x-1) + 3$

b)  $g(x) = f(-x)$

c)  $p(x) = \frac{1}{2}f(x) - 2$

d)  $r(t) = f(2x)$

12. Given the functions  $f(x) = \frac{x}{x+3}$  and  $g(x) = \frac{5}{x}$ , find and simplify  $f \circ g$  and  $g \circ f$ . Determine their domains.

13. The function  $f$  is one-to-one. Find the inverse function and check your answer. Determine the domain and range for the function and its inverse.

a)  $f(x) = \frac{2x+3}{x+2}$

b)  $f(x) = 3 - \sqrt[3]{x-5}$

14. Use synthetic division to divide  $4x^6 + 3x^4 + x^2 + 5$  by  $x + 2$ .

15. List the potential rational zeros of the polynomial function  $p(x) = 4x^6 + 3x^4 + x^2 + 5$

16. Find all real zeros of the given polynomial function. Express the function in a factored form.

$$P(x) = 3x^4 + 4x^3 + 7x^2 + 8x + 2$$

17. Find the bounds on the real zeros of  $p(x) = 4x^6 + 3x^4 + x^2 + 5$

18. Show that  $(x - 3)^2$  is a factor of the polynomial  $p(x) = x^4 - 3x^3 - 19x^2 + 87x - 90$ .

19. Consider the polynomial

$$f(x) = 5x^4 - 4x^3 + 19x^2 - 16x - 4$$

a) What is the maximum number of zeros that  $f$  can have?

b) List the possible rational zeros of  $f$

c) Use the Remainder Theorem to find all the zeros of  $f$  and use the synthetic division to aid in factoring  $f$  completely.

20. Function  $p(x) = 2x^3 + x^2 + 2x + 1$  is given

a) List real zeros and multiplicity

b) Determine whether the graph crosses or touches the  $x$ -axis at the  $x$ -intercept.

c) Determine the end behavior.

d) Sketch the graph of  $p(x)$

21. Let  $p(x) = -2x^3(x-3)(x+4)^2(x^2+5)$

a) List real zeros and their multiplicity

b) Determine whether the graph crosses or touches the  $x$ -axis at each  $x$ -intercept.

c) Determine the end behavior of  $p$ .

d) Sketch the graph of  $p(x)$

22. Sketch the graph of the given rational function by finding the domain,  $x$ -intercept(s),  $y$ -intercept, asymptotes (vertical, horizontal, oblique), checking for symmetry, finding the points where the graph crosses the horizontal/oblique asymptote and constructing the sign chart to determine where the graph is above/below the  $x$ -axis.

a)  $f(x) = \frac{3x^2 - 3x}{x^2 + x - 12}$

b)  $f(x) = \frac{x^2 - x - 12}{x + 1}$

23. Find the vertical, horizontal or oblique asymptotes, if any, of the rational function

a)  $f(x) = \frac{2x^2 - 5x + 2}{x^2 - 4}$

b)  $f(x) = \frac{x^3 + 3x^2 + 5}{x^2 + 3}$

c)  $f(x) = \frac{x^2 - 1}{x^2 + x - 2}$

24. Solve the inequality; write the answer in the interval notation

a)  $x^3 + 4x^2 \geq x + 4$

b)  $\frac{-2}{1-3x} < 1$

c)  $\frac{x(x^2 + x - 2)}{x^2 + 9x + 20} \leq 0$

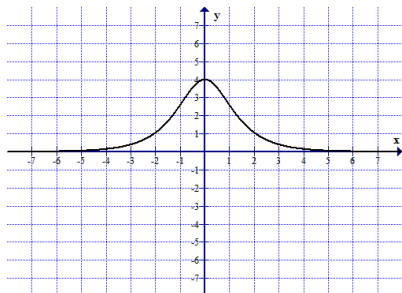
d)  $\frac{5}{x+2} \geq \frac{3}{x+1}$

25. Let  $f(x) = |x - 2|$  and  $g(x) = \frac{2}{x+1}$ . Find a)  $(f \circ g)(4)$  b)  $(g \circ f)(2)$  c)  $(f \circ f)(1)$  d)  $(g \circ g)(0)$

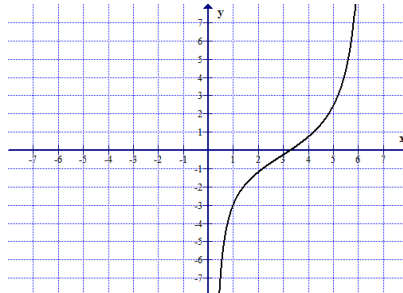
25'. Find functions  $f$  and  $g$  so that  $f \circ g = H$ , where  $H(x) = \sqrt{x^2 + 3x - 2}$ .

26. Determine whether the following functions are one-to-one. Explain

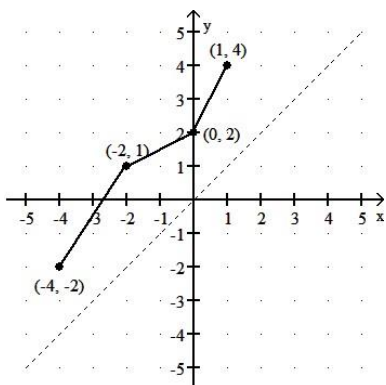
a)



b)



27. Use the graph of the given one-to-one function to sketch the graph of the inverse function.



28. Change an exponential expression to an equivalent expression involving a logarithm

a)  $2 \cdot 2^N = 5$

b)  $e^x = 8$

29. Change each logarithmic expression to an equivalent expression involving an exponent

a)  $\log_b 4 = 2$

b)  $\ln x = 4$

30. Find the exact value of each logarithm without using a calculator

a)  $\log_{1/2} 4$     b)  $\log_3 \frac{1}{27}$     c)  $\ln e^3$     d)  $\log_5 5^{4.2}$

31. Use transformation to graph  $f(x) = f(x) = -3^{-x+1} + 2$ . Plot accurately at least 3 points on the graph of a basic function and use them to perform the transformations. Determine domain, range and the equation of horizontal asymptote

32. Find the domain of the given functions

a)  $f(x) = \log_7\left(\frac{x+1}{x}\right)$

b)  $f(x) = x + \log_7\left(\frac{x+1}{9-x^2}\right)$

33. Given  $f(x) = -\ln(2x+4)$

a) Find the domain of  $f$

b) Use transformations to graph  $f$

c) Use the graph to find the range and any asymptotes of  $f$

d) Find the inverse function  $f^{-1}$

e) Graph the inverse function

34. Let  $f(x) = 2 + \log_3(x+1)$ .

a) Use transformations to graph function  $f$ .

b) Use the graph to find the range of  $f$  and any asymptotes

35. (a) Rewrite the expression as a single logarithm and simplify

$$8 \log_2 \sqrt{3x-2} - \log_2 \left(\frac{4}{x}\right) + \log_2 4$$

(b) Express as a single logarithm. Assume all variables are positive and  $b \neq 1$

$$2 \log_b t - \frac{4}{5} \log_b s + \frac{1}{3} \log_b v - 4 \log_b u$$

36. Rewrite the expression as a sum/difference of logarithms. Express powers as factors.

a)  $\ln\left(\frac{7x^3\sqrt{1-6x}}{(x+1)(x-3)^2}\right)$

b)

$$\ln\left(\frac{(x-2)^2}{x^4\sqrt{x-3}}\right)$$

37. Use properties of logarithms to find the exact value of each expression

a)  $\log_6 9 + \log_6 4$

b)  $2^{\log_2 5}$

c)  $\log_2 6 \cdot \log_6 4$

38. Solve the equations

a)  $2^{-x+1} = 5^{1-2x}$

b)  $\log_3 x + \log_3(x-2) = \log_3(x+4)$

c)  $\ln(x+1) - \ln x = 2$

d)  $5(2^{3x}) = 8$

e)  $3^{2x} + 3^x - 2 = 0$

f)  $4^{x^2+4x} = \frac{1}{64}$

g)  $\log_2(x+4) + \log_2(x+3) = 1$

h)  $5e^x - 2 = 23$

i)  $\log_3(x^2 + x + 7) = 2$

j)  $(e^4)^x e^{x^2} = e^{12}$

k)  $\log_x 81 = 4$

l)  $e^{2x+5} = \frac{1}{3}$

39. Use the Change-of-the base formula and a calculator to approximate  $\log_7 5$ .

40. Find an equation of the parabola with given properties. Graph the equation.

a) Vertex at (0, 0); axis of symmetry the x-axis; containing the point (4, 9)

b) Focus at (-2, 0); directrix the line  $x = 2$

c) Focus at (-3,-2); directrix the line  $y = 2$

41. Find the vertex, focus, directrix, the end points of the latus rectum segment of the parabola given by the equation  $y^2 + 12y = -2x - 30$ . Graph the parabola.

42. Find the equation of the ellipse with given properties. For each ellipse find the center, foci, and vertices. Find the lengths of major and minor axes.

a) Focus at (-6,0); vertices  $(\pm 7, 0)$ .

b) Foci at (1,2) and (-3,2); vertex at (-4,2)

c) Center at (2,-1); vertex at (7,-1); focus at (4,-1)

43. Find the center, foci, and vertices of the ellipse  $4x^2 + 9y^2 + 8x - 18y = 23$ . Graph the ellipse.

44. Find the equation of the hyperbola with given properties. For each hyperbola find the center, vertices, foci, equations of transverse and conjugate axis, and the equations of the asymptotes.

a) vertices at  $(\pm 4, 0)$ ; focus at (5,0)

b) Foci at  $(0, \pm 2)$ ; asymptote the line  $y = -x$

c) Focus at (-4,0); vertices at (-4,4) and (-4,2)

45. Graph the conic

$$4x^2 + 25y^2 + 24x - 50y - 39 = 0$$

46. Identify the graph of a given equation as a circle, ellipse, parabola, hyperbola, not a conic.

a)  $2x + 3y = 6$

b)  $2x^2 - 5y^2 + 7x - 6y - 4 = 0$

c)  $4y^2 + 2x - 4y + 1 = 0$

d)  $3x^2 + 7y^2 - 2x + 3y - 2 = 0$

e)  $x^2 + y^2 - 5x - 11y + 2 = 0$

47. Solve the following system of equations. If there are no solutions, say so. If there are infinitely many solutions, describe the solution set.

a) 
$$\begin{cases} 5x - y = 13 \\ 2x + 3y = 12 \end{cases}$$

$$b) \begin{cases} 2x + y = 1 \\ 4x + 2y = 3 \end{cases}$$

$$c) \begin{cases} x + 2y = 4 \\ 2x = 8 - 4y \end{cases}$$

$$d) \begin{cases} xy = 4 \\ 2x^2 - xy + y^2 = 8 \end{cases}$$

48. Solve the linear system by elimination

$$\begin{cases} x - y - z = 1 \\ -x + 2y - 3z = -4 \\ 3x - 2y - 7z = 0 \end{cases}$$

49. Evaluate the determinants

$$a) \begin{vmatrix} -3 & -1 \\ 4 & 2 \end{vmatrix}$$

$$b) \begin{vmatrix} 4 & -1 & 2 \\ 6 & -1 & 0 \\ 1 & -3 & 4 \end{vmatrix}$$

50. Solve the system using Cramer's Rule. If Cramer's Rule is not applicable, say so.

$$a) \begin{cases} 3x - 6y = 24 \\ 5x + 4y = 12 \end{cases}$$

$$b) \begin{cases} 3x - 2y = 4 \\ 6x - 4y = 0 \end{cases}$$

$$c) \begin{cases} x + y - z = 6 \\ 3x - 2y + z = -5 \\ x + 3y - 2z = 14 \end{cases}$$

$$d) \begin{cases} x - y + 2z = 5 \\ 3x + 2y = 4 \\ -2x + 2y - 4z = -10 \end{cases}$$

51. Graph each equation of the system and solve it to find points of intersection

$$a) \begin{cases} xy = 3 \\ x^2 + y^2 = 10 \end{cases}$$

$$b) \begin{cases} 2y - x = 2 \\ x^2 - 4y^2 = 16 \end{cases}$$

52. Write down the first five terms of the sequence



a)  $\{a_n\} = \left\{ \frac{3^n}{n} \right\}$

b)  $\{d_n\} = \left\{ (-1)^n \left( \frac{n}{2n-1} \right) \right\}$

c)  $a_1 = -1, \quad a_n = n + 3a_{n-1}$

53. Write the formula for the  $n^{\text{th}}$  term of a sequence  $\{a_n\}$  given by

a)  $-1, 2, -3, 4, -5, 6, \dots$

b)  $\frac{1}{1 \cdot 2}, \frac{1}{2 \cdot 3}, \frac{1}{3 \cdot 4}, \frac{1}{4 \cdot 5}, \dots$

54. Write out each sum

a)  $\sum_{k=0}^n \frac{1}{3^k}$

b)  $\sum_{k=1}^n \frac{k^2}{2}$

c)  $\sum_{j=5}^{10} (-1)^j (2j + 3)$

55. Express each sum using summation notation. Use 1 as the lower limit of summation and  $i$  for the index of summation.

a)  $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{13}{14}$

a)  $3^2 + 6^3 + 9^4 + \dots + 24^9 + \dots$

b)  $1 - \frac{1}{3} + \frac{1}{9} - \frac{1}{27} + \dots + \frac{1}{729}$

56. Find the sum

a)  $\sum_{k=1}^{40} k$

b)  $\sum_{k=1}^{52} (3k - 7)$

c)  $\sum_{k=1}^{50} (2k + 1)^2$

57. Show that the sequence  $\{a_n\} = \{3n + 7\}, n \geq 1$  is arithmetic. Find the first term and the common difference

58. Find the 80<sup>th</sup> term of the arithmetic sequence  $1, -2, -5, \dots$

59. Find the  $n^{\text{th}}$ -term of the arithmetic sequence if

a)  $a_1 = -2, \quad d = 4$ ;  $d$  is the common difference

b) 5<sup>th</sup> term is  $-2$  and the 13<sup>th</sup> term is  $30$  (You must use a system of equations to solve this problem).

60. Consider the finite arithmetic sequence:  $73, 78, 83, 88, \dots, 558$

a) Write the  $n^{\text{th}}$  term by finding the first term and the common difference.

b) Write the sum of all its terms in sigma form.

c) Find the value of the sum by using an appropriate formula.

61. Find the sum of the first 38 terms of the arithmetic sequence 3, 8, 13, 18, 23, ...

62. Find  $x$  so that  $x + 3$ ,  $2x + 1$  and  $5x + 2$  are consecutive terms of an arithmetic sequence.

63. Show that the sequence  $\{a_n\} = \left\{ \frac{2^n}{3^{n-1}} \right\}$  is geometric. Find the first term and the common ratio.

64. Find the fifth and the  $n$ -th term of the geometric sequence for which  $a_1 = -2$ ,  $r = -1/3$

65. Find the 10-th term of the geometric sequence -1, 2, -4, ...

66. Find the common ratio and the formula for the  $n$ -th term of the geometric sequence given

a)  $a_{10} = 786432$ ,  $r = -2$

b)  $a_5 = 32/81$ ,  $a_8 = 256/2187$

67. Consider the geometric sequence:  $2, \frac{6}{5}, \frac{18}{25}, \dots, 2\left(\frac{3}{5}\right)^{n-1}, \dots$

a) Write the formula for the  $n$ -th term.

b) Write the sum of the first  $n$  terms of this sequence using sigma notation.

c) Find the formula for the sum of the first  $n$  terms of this sequence.

d) Find the sum of the first 10 terms of this sequence

68. Determine whether each geometric series converges or diverges. Explain. If a series converges, find its sum.

a)  $\sum_{k=1}^{\infty} 10(1.06)^{k-1}$     b)  $\sum_{k=1}^{\infty} 1000\left(-\frac{2}{3}\right)^{k-1}$     c)  $2 - \frac{1}{2} + \frac{1}{8} - \frac{1}{32} + \dots$     d)  $8 + 12 + 18 + 27 + \dots$

69. Evaluate without using a calculator

a)  $5!$

b)  $\binom{12}{3}$

70. Use the Binomial Theorem to expand given power of a binomial

a)  $(3x + 2)^7$

b)  $(x - 2y)^5$

71. Find the coefficient of  $x^4$  in the expansion of

a)  $(4x + 1)^7$

b)  $\left(x - \frac{2}{\sqrt{x}}\right)^{10}$

72. Find the 7-th term of  $(2x + 3)^9$ .

73. Prove by mathematical induction that for all natural numbers  $n \geq 1$  :

a)  $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$

b)  $\sum_{k=1}^n 3^k = \frac{3(3^n - 1)}{2}$

c)  $n(n+2) < (n+1)^2$

d) 2 is a factor of  $n^2 + n$

e)  $3 + 8 + 13 + \dots + (5n - 2) = \frac{n(5n + 1)}{2}$

## Answers

1.

- a)  $(3x+2)(x+4)$
- b)  $(x^2+1)(x-1)(x+1)$
- c)  $-2(2x-1)(2x+1)(x^2+2)$
- d)  $(x+1)(x^2-x+1)(x-1)$
- e)  $3(x^2+5)(x-\sqrt{5})(x+\sqrt{5})$
- f)  $(x+1)(x-1)(x-7)$
- g)  $(2x+5)(4x^2-10x+25)$
- h)  $(7x-16)(x-3)$
- i)  $2(x-1)^2(x+4)^6(-x+9)$

2.

- a) Domain:  $(-\infty, -4) \cup (-4, -1/2) \cup (-1/2, \infty)$
- b)  $(0, 1]$
- c) All real numbers
- d)  $[-3, -2) \cup (-2, -2) \cup (2, \infty)$
- e)  $(-4, 1] \cup (4, \infty)$

3.

- a)  $f(0) = 0$
- b)  $f(-x) = \frac{-x}{x^2+1}$
- c)  $f(x+h) = \frac{x+h}{x^2+2xh+h^2+1}$
- d)  $f(2x) = \frac{2x}{4x^2+1}$
- e)  $f(-3+h) = \frac{-3+h}{h^2-6h+10}$
- f)  $\frac{-2h^2-3h}{5h^2+20h+25}$

4.

- a)  $\frac{3}{(x-4)(x+h-4)}$
- b)  $(-4x - 2h + 1)$

5. Function, domain

- a)  $\frac{2}{x} + \sqrt{x+1}, [-1, 0) \cup (0, \infty)$
- b)  $\frac{2}{x} - \sqrt{x+1}, [-1, 0) \cup (0, \infty)$
- c)  $\frac{2\sqrt{x+1}}{x}, [-1, 0) \cup (0, \infty)$
- d)  $\frac{2}{x\sqrt{x+1}}, (-1, 0) \cup (0, \infty)$

6. B and D represent functions because they pass the vertical line test

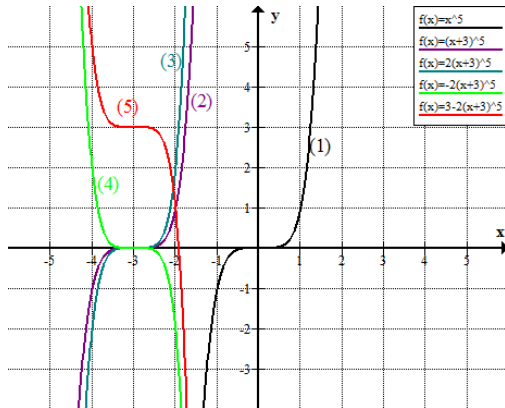
7. Based on the graph:

- a)  $f(0) = -2$  and  $f(-4) = -6$
- b) Negative
- c)  $(-\infty, 3]$
- d)  $[-8, \infty)$

- e)  $x = 2, -1, \text{ and } -5$
- f)  $y = -2$
- g)  $x = -2, -4$
- h)  $(-3, -1) \text{ and } (0.5, 3)$
- i)  $(-\infty, -3) \text{ and } (-1, 0.5)$
- j)  $(-\infty, -5) \text{ and } (2, 3]$
- k)  $(-5, -1) \text{ and } (-1, 2)$
- l) Neither

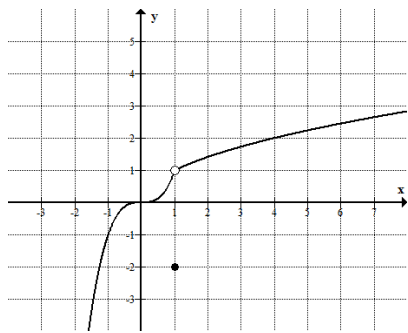
8. Odd

9.  $f(x) = 3 - 2(x + 3)^5$

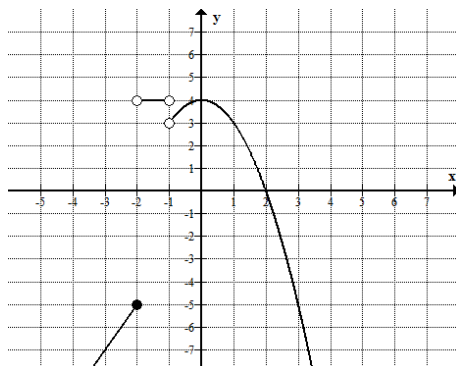


10. Piecewise functions:

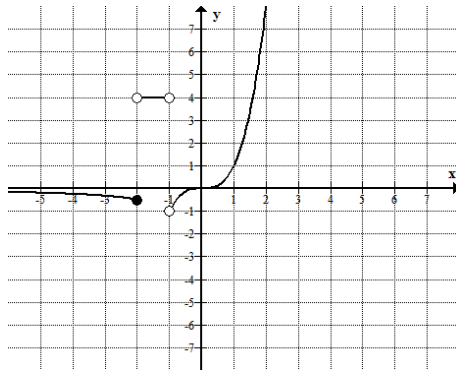
a)



b)



c)



11. (Graph)

- Shift 1 right, up 3
- Reflect about y-axis
- Compress vertically by a factor of 2 and shift down by 2 units
- Compress horizontally by a factor of 2

12.

- $f \circ g(x) = \frac{5}{5+3x}$
- $g \circ f(x) = \frac{5x+15}{x}$

13. Inverse:

- $f^{-1}(x) = \frac{-2x+3}{x-2}$  Domain:  $(-\infty, 2) \cup (2, \infty)$ , Range:  $(-\infty, -2) \cup (-2, \infty)$
- $f^{-1}(x) = (-x + 3)^3 + 5$  Domain: All real numbers, Range: All real numbers

14.  $4x^5 - 8x^4 + 19x^3 - 38x^2 + 77x - 154 + \frac{313}{x+2}$

15. Potential rational zeroes:  $\frac{p}{q} = \pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 5, \pm \frac{5}{2}, \pm \frac{5}{4}$

16. Real zeroes:  $-1, -\frac{1}{3}$ , factored:  $(3x + 1)(x + 1)(x^2 + 2)$

17. Between  $-6$  and  $6$

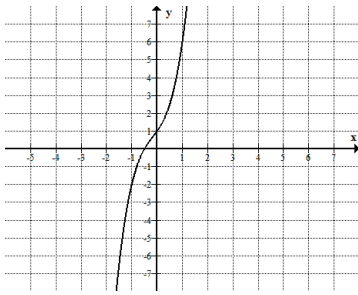
18.  $(x - 3)^2(x^2 + 3x - 10)$

19.

- 4 (the number of zeros cannot exceed the degree)
- $\pm 1, \pm 2, \pm 4, \pm \frac{1}{5}; \pm \frac{2}{5}; \pm \frac{4}{5}$
- Zeros:  $1, -1/5$ ; factored:  $(5x + 1)(x - 1)(x^2 + 4)$

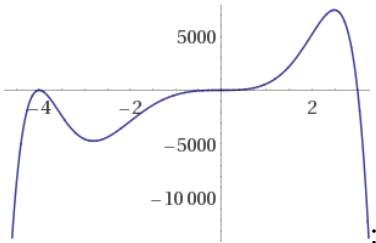
20.

- Zeros:  $-\frac{1}{2}$ , multiplicity 1
- Crosses at  $-\frac{1}{2}$
- $y = 2x^3$
-



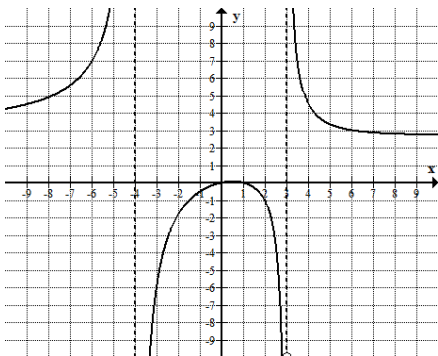
21.

- a)  $x=0$  multiplicity 3;  $x=3$  multiplicity 1,  $x=-4$  multiplicity 2
- b) Crosses at  $x=0, 3$ , and touches at  $x = -4$
- c)  $y = -2x^8$
- d)

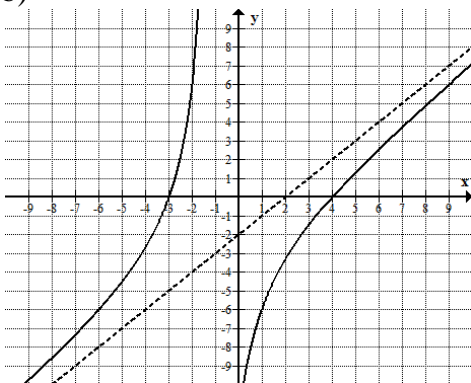


22. Graph

a)



b)



23. Find asymptotes:

- a) Vertical  $x = \pm 2$ , horizontal  $y = 2$
- b) No vertical or horizontal asymptotes. Oblique asymptote  $y = x + 3$

c) Vertical  $x = -2$ , horizontal  $y = 1$

24.

a)  $[-4, -1] \cup [1, \infty]$

b)  $(1/3, 1)$

c)  $(-\infty, -2) \cup [0, 1] \cup (4, 5)$

d)  $(-2, -1) \cup [\frac{1}{2}, \infty)$

25. Given  $f(x)$  and  $g(x)$ :

a)  $(f \circ g)(4) = \frac{8}{5}$

b)  $(g \circ f)(2) = 2$

c)  $(f \circ f)(1) = 1$

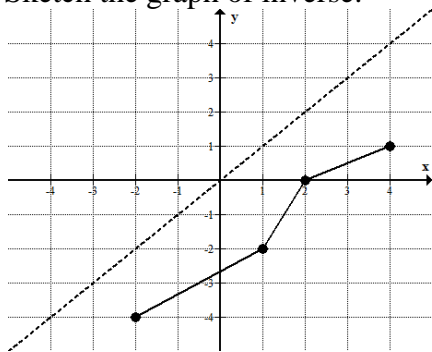
d)  $(g \circ g)(0) = \frac{2}{3}$

25'.  $f(x) = \sqrt{x}$ ;  $g(x) = x^2 + 3x - 2$

26. a) No – fails horizontal line test

b) Yes – passes horizontal line test

27. Sketch the graph of inverse:



28.

a)  $\log_{2.2} 5 = N$

b)  $\log_e 8 = x$  or  $\ln 8 = x$

29.

a)  $b^2 = 4$

b)  $e^4 = x$

30.

a)  $-2$

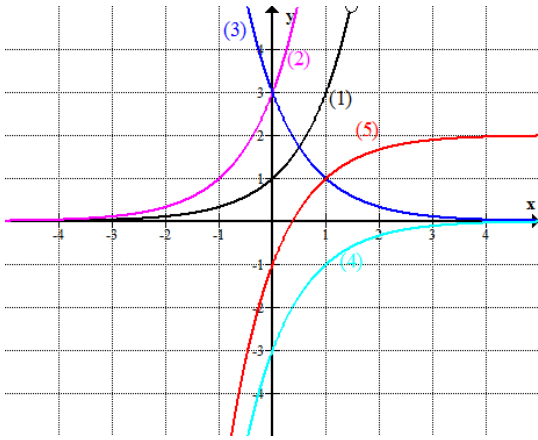
b)  $-3$

c)  $3$

d)  $4.2$

31. Domain: all real numbers; Range:  $y < 2$  (asymptote at  $y=2$ )





32. Domain:

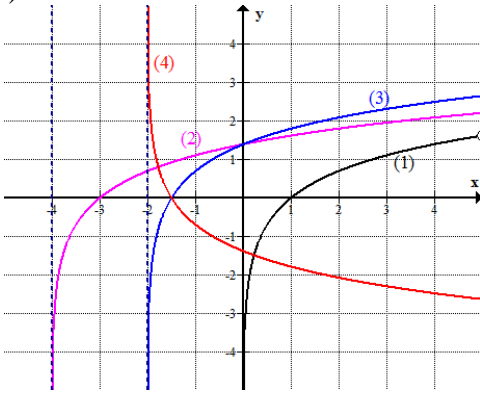
a)  $(-\infty, -1) \cup (0, \infty)$

b)  $(-\infty, -3) \cup (-1, 3)$

33.

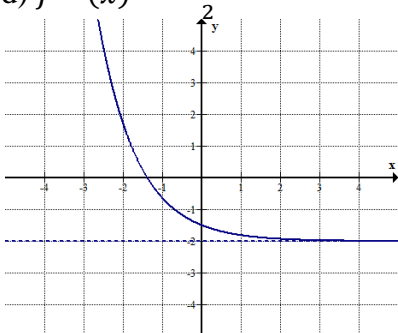
a) Domain:  $(-2, \infty)$

b)



c) Range: all real numbers; asymptote  $x = -2$

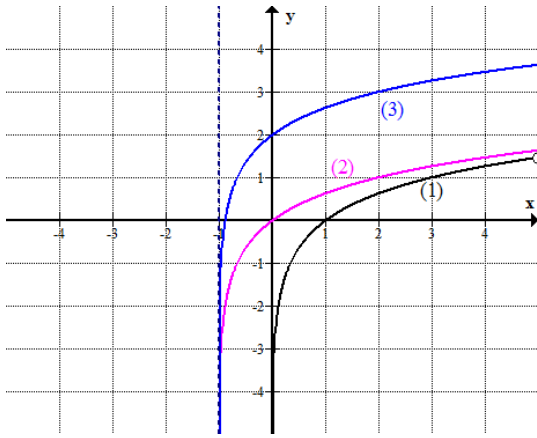
d)  $f^{-1}(x) = \frac{e^{-x}-4}{2}$



e)

34

a)



b) Range: all real numbers, vertical asymptote:  $x = -1$

35.

a)  $\log_2[x(3x - 2)^4]$

b)  $\log_b \frac{t^2 v^{1/3}}{s^4/5u^4}$

36.

a)  $\ln 7 + \ln x + \frac{1}{3} \ln(1 - 6x) - \ln(x + 1) - 2 \ln(x - 3)$

b)  $2 \ln(x - 2) - 4 \ln x - \frac{1}{2} \ln(x - 3)$

37.

a) 2

b) 5

c) 2 (using change of base formula)

38.

a)  $\frac{\log_2^5}{\log 50} \approx 0.234$

b) 4

c)  $1/(e^2 - 1)$

d)  $\frac{\ln^8}{3 \ln 2}$

e)  $x = 0$  (use substitution)

f)  $x = -3$  and  $x = -1$  (2 solutions)

g)  $x = -2$

h)  $x = \ln 5 \approx 1.61$

i)  $x = -2$  and  $x = 1$

j)  $x = -6$  and  $x = 2$

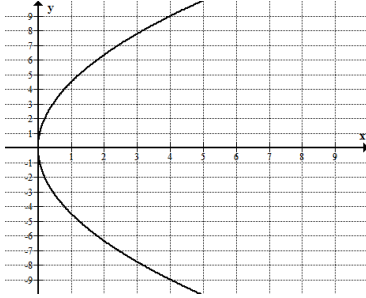
k)  $x = 3$

l)  $x = \frac{\ln^{\frac{1}{3}} - 5}{2}$

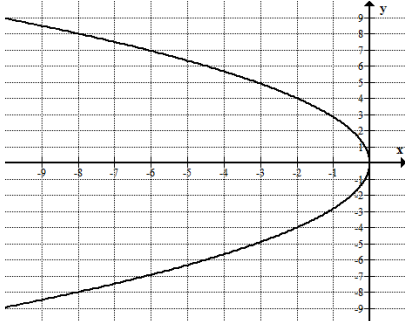
39.  $\frac{\ln 5}{\ln 7} \approx .8271$

40.

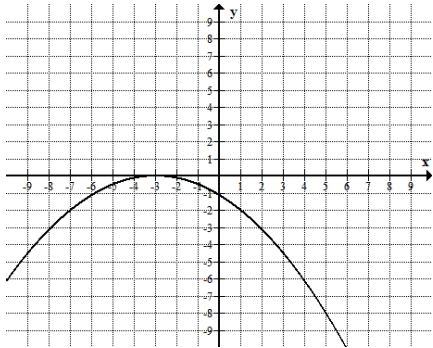
a)  $y^2 = \frac{81}{4}x$



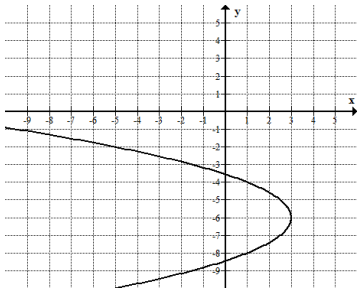
b)  $y^2 = -8x$



c)  $(x+3)^2 = -8y$



41. Equation:  $(y + 6)^2 = -2(x - 3)$ , vertex:  $(3, -6)$ , focus:  $(\frac{5}{2}, -6)$ , directrix line :  $x = 7/2$



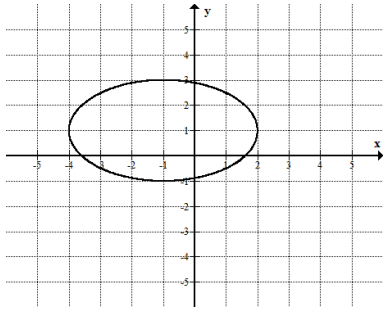
42.

a)  $\frac{x^2}{49} + \frac{y^2}{13} = 1$

b)  $\frac{(x+1)^2}{9} + \frac{(y-2)^2}{5} = 1$

c)  $\frac{(x-2)^2}{25} + \frac{(y+1)^2}{21} = 1$

43.



Center:  $(-1, 1)$ , Foci  $(-1 - \sqrt{5}, 1)$ ,  $(-1 + \sqrt{5}, 1)$ , Vertices:  $(-4, 1)$  and  $(2, 1)$

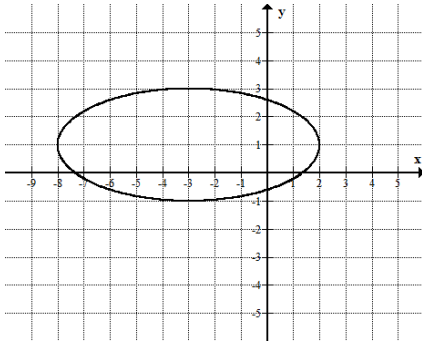
44.

a)  $\frac{x^2}{16} - \frac{y^2}{9} = 1$

b)  $\frac{y^2}{2} - \frac{x^2}{2} = 1$

c)  $\frac{(y-3)^2}{1} - \frac{(x+4)^2}{8} = 1$

45.



46.

- a) Not a conic
- b) Hyperbola
- c) Parabola
- d) Ellipse
- e) Ellipse (in fact a circle)

47. a)  $x = 3, y = 2$

b) No solution

c) Infinitely many solutions  $\{(x, y) \mid x = -2y + 4\}$

d)  $(2, 2), (-2, -2), (\sqrt{2}, 2\sqrt{2}), (-\sqrt{2}, -2\sqrt{2})$

48. Infinitely many solutions  $\left\{ (x, y, z) \mid y = \frac{4}{5}x - \frac{7}{5}, z = \frac{1}{5}x + \frac{2}{5}, x \text{ is any real number} \right\}$

49. Determinants

a)  $-2$

b)  $-26$

50. a)  $(4, -2)$

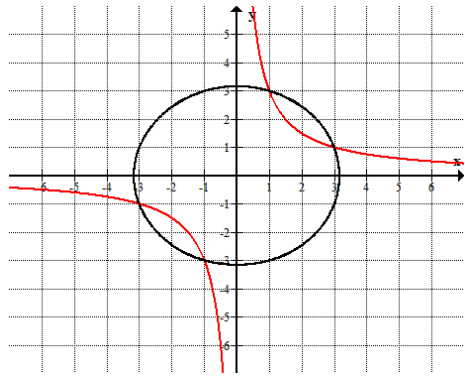
b) Cramer's Rule is not applicable because the determinant is 0

c)  $(1, 3, -2)$

d) Cramer's Rule is not applicable because the determinant is 0.

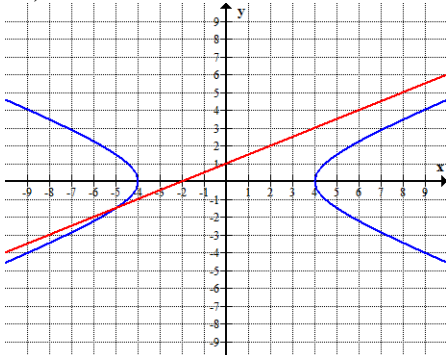
51.

a)



(1, 3), (3, 1), (-1, -3), and (-3, -1)

b)



$(-5, -3/2)$

52.

a)  $3, \frac{9}{2}, 9, \frac{81}{4}, \frac{243}{5}$

b)  $-1, \frac{2}{3}, -\frac{3}{5}, \frac{4}{7}, -\frac{5}{9}$

c)  $-1, -1, 0, 4, 17$

53.

a)  $(-1)^n \times n$

b)  $\frac{1}{n \times (n+1)}$

54.

a)  $\frac{1}{3^0} + \frac{1}{3^1} + \frac{1}{3^2} + \dots + \frac{1}{3^n}$

b)  $\frac{1^2}{2} + \frac{2^2}{2} + \frac{3^2}{2} + \dots + \frac{n^2}{2}$

c)  $(-1)^5(2(5) + 3) + (-1)^6(2(6) + 3) + (-1)^7(2(7) + 3) + (-1)^8(2(8) + 3) + (-1)^9(2(9) + 3) + (-1)^{10}(2(10) + 3)$

55.

a)  $\sum_{k=1}^{13} \frac{k}{k+1}$

b)  $\sum_{k=1}^n (3k)^{k+1}$

c)  $\sum_{k=1}^7 (-1)^{k+1} \frac{1}{3^{k-1}}$

56.

a) 820

b) 3770

c) 176850

57.  $a_1 = 10$ , common difference  $d = 3$

58.  $1 - 3(n - 1)$ ; 80<sup>th</sup> term:  $-236$

59.

a)  $a_n = -2 + 4(n - 1)$

b)  $a_n = -18 + 4(n - 1)$

60.

a)  $a_n = 68 + 5n$

b)  $\sum_{k=1}^{98} (68 + 5k)$

c)  $\sum_{k=1}^{98} (68 + 5k) = 30919$

61.  $\sum_{k=1}^{38} (3 + 5(k - 1)) = 2223$

62.  $x = -5/2$

63.  $a_1 = 2$ , common ratio  $r = \frac{2}{3}$ ; sequence:  $2, \frac{4}{3}, \frac{8}{9}, \frac{16}{27}, \frac{32}{81}, \dots$

64.  $x_5 = \frac{-2}{81}$ ,  $x_n = (-2)\left(-\frac{1}{3}\right)^{n-1}$

65.  $x_{10} = (-1)(-2)^9 = -512$

66.

a) Common ratio  $r = -2$ ;  $a_n = (3)(-2)^{n-1}$

b) Common ratio  $r = \frac{2}{3}$ ,  $a_n = 2\left(\frac{2}{3}\right)^{n-1}$

67.

a)  $a_n = (2)\left(\frac{3}{5}\right)^{n-1}$

b)  $\sum_{k=1}^n (2)\left(\frac{3}{5}\right)^{k-1}$

c)  $S_n = \frac{(2)\left(1 - \left(\frac{3}{5}\right)^n\right)}{1 - \frac{3}{5}}$

d)  $5\left(1 - \frac{3^{10}}{5^{10}}\right)$

68.

a) Diverges

b) Converges, sum = 600

c) Converges, sum = 1.6

d) Diverges

69.

a) 120

b) 220

70.

$2187x^7 + 10206x^6 + 20412x^5 + 22680x^4 + 15120x^3 + 6048x^2 + 1344x + 128$

a) (8 terms)

$$x^5 - 10x^4y + 40x^3y^2 - 80x^2y^3 + 80xy^4 - 32y^5$$

b) (6 terms)

71.

a) 8960

b) 336

72.  $145152x^6$