

PDE & ODE Qualifying Examination

ODE course: MAP 5316

Description: Existence and uniqueness theorem; stability and the direct method of Lyapunov; invariant manifolds; limit sets and the Poincare-Bendixon theory; linear systems and stability of nonlinear systems; hyperbolic theory.

Prerequisites: MAA 3200, MAA 4402 and MAS 3105.

Textbooks:

- Carmen Chicone, Ordinary Differential Equations with Applications, 2006, Springer-Verlag, New York, 2nd edition.
- Walter Wolfgang, Ordinary Differential Equations, Springer Science & Business Media, 1998.

Chapters covered (from C. Chicone): 1.1–1.8, 1.9, 1.10; 2.1–2.3; 3.6; 4.1–4.3.

A good source of problems with solutions is the textbook by Steven H. Strogatz, "Non-linear Dynamics And Chaos: With Applications To Physics, Biology, Chemistry And Engineering", 2nd edition, ISBN: 0813349109.

PDE course: MAP 6326

Description (catalog): Basic concepts of first and second order PDE's applications to optics and wave fronts, Cauchy problems, Laplace equation, Green's function, Dirichlet problem, heat equation. Prerequisite: MAA 4211.

Textbooks:

- Walter A. Strauss, Partial Differential Equations: An Introduction, John Wiley & Sons, 2007, 2nd edition.
- Fritz John, Partial Differential Equations, 4th Edition, Applied Mathematical Sciences, Vol. 1, Springer Verlag, 1995.
- L. C. Evans, Partial Differential Equations, 2nd Edition, Graduate Series in Mathematics, vol. 19.R, AMS, 2010.

Chapters covered (from W.A. Strauss): 1–7; 9.2; 12.1–12.3; 14.1;

Chapters covered (from L.C. Evans): Chapter 2 (Section 2.2: Laplace equation; Section 2.3: The heat equation; Section 2.4: The wave equation).

- Main topics**
- 1st-order linear, quasi-linear and non-linear PDE's using the method of characteristics: know how to obtain explicit solutions.
 - Classification of 2nd-order linear equations in two independent variables: hyperbolic, parabolic and elliptic types.
 - Power series solutions and the Cauchy-Kovalevski theorem.
 - The wave equation: explicit formulas for the initial-value problem in dimensions 1, 2 and 3; energy and uniqueness; Fourier series solutions; Duhamel's principle; Huygens' principle (sharp signals).
 - The Laplace equation: Green's identities, mean value theorem, maximum principle, fundamental solution.
 - The Dirichlet (Neumann) problem and Poisson formula.
 - The heat equation: Fourier series solutions, maximum principle, Gaussian kernel for the pure initial value problem.

Timeline for Completion The written graduate PDE & ODE examination will have two parts/sections with much of the weight (>65%) being placed on the PDE component; on the first, the student will be tested on material covered from MAP 5316, while on the second part, most of the material is covered in MAP 6326. Ordinarily first-year students take the aforementioned courses (if necessary) throughout the year to prepare them for the examination. Students can take the qualifying examination during their second year. Students are given four hours to complete the qualifying examination. The latter will mainly consist of two parts (ODE part+PDE part). A list of practice questions (which is not meant to be exhaustive) along with one sample exam is provided.

Attempts Unless explicitly permitted by the Graduate Director, every student may attempt a qualifying examination only after both related courses are completed for a grade. Incoming students with advanced backgrounds, with the approval of the Graduate Director, may attempt the qualifying examination without taking the associated sequence of courses. In these cases only, a result of Fail is not counted against the number of attempts (= 2) allowed and the student will be advised to take the appropriate course before attempting the qualifying examination in the following semester.

Results There are three possible results from an attempt: Ph.D. level pass, Partial pass and Fail. Partial passes on material in one of the two core courses (ODE+PDE) may be awarded. A student is required to retake only the part of the examination that was failed. In the case when the ODE part of the examination is retaken the amount of time allowed is 1/3 of that allowed for the full examination. In the case when the PDE part of the examination is retaken the amount of time allowed is 2/3 of that allowed for the full examination. Students may check their graded qualifying exams with the Graduate Director or the appropriate faculty member. Copies of the qualifying exam may not be made. Students are expected to retake any failed examinations in the next offering of the examination. Two attempts per qualifying examination are allowed.