Geometry/Topology Qualifying Exam
Syllabus

This exam covers the material of a standard first year course in Geometry and Topology that include:

- point set topology and introductory algebraic topology (covering spaces), and
- differentiable manifolds.

Below is the material from these courses and suggested texts that are relevant for the preparation for this exam.

(1) Point set topology / Algebraic topology:

Suggested texts:

1. Munkres, *Topology*, 2nd edition. The exam will cover material from Sections 1-7,12-24,26-28,30-36,43,45-47, 51-55,58,59,79-82.

2. Greenberg and Harper, *Algebraic topology: a first course*. The exam will cover material from Part I.

3. Hatcher, *Algebraic Topology*, Chapter 1.

Suggested course: MTG 4302 Topology. Students who are already familiar with the material from point set topology described below may prepare for this exam by taking MTG 5326 to cover the material from algebraic topology.

Topics included in Qualifying Exam
(from point set topology/introductory algebraic topology):

1. *Basic set theory:* Relations, equivalent relations, equivalence classes, order relations. Countable and uncountable sets.

2. *Definitions and examples:* Metric spaces. Topological spaces and continuous maps. Basis and subbasis. The product topology, the order topology, the finite-complement topology. Subspace and quotient topology. The metric topology. Homeomorphisms.

3. *Properties of topological spaces:* Connectness, path-connectedness. Compactness. Sequential compactness and limit point compactness. Countability axioms, separation axioms (Hausdorff, regular, normal).

4. *Famous theorems:* Uryshohn lemma, Urysohn Metrization theorem, Tietze Extension theorem.

5. *Topology and metric spaces:* Complete metric spaces, compactness in metric spaces, Ascoli's theorem.

6. *Introduction to algebraic topology:* Homotopy of maps. Fundamental group and its dependence on basepoint and functoriality. Retractiosn and fixed points. Covering spaces, their relation to the fundamental group. Existence and uniqueness of lifts.

(2) Differential Geometry

Suggested text:

1. Jeffrey Lee, *Manifolds and Differential Geometry,* Graduate Studies in Math., AMS v. 107 (2009)

Suggested course: MTG5256

Topics included in Qualifying Exam:

1. Smooth manifolds, submanifolds, tangent vectors and vector fields. Flow of a vector field, commutator of vector fields.
2. Tensors on a vector space, tensors on a manifold. Symmetric and skew-symmetric tensors space and differential forms on a manifold. Metric and symplectic form.
3. Differentiation of tensors, Lie derivatives, differential, Cartan identities.
4. Orientation and integration on manifolds. Stokes Theorem, Divergence Theorem on manifolds.