Syllabus for Algebra Qualifying Exam
FIU Math Dept

Suggested References

Aluffi, P. Algebra: Chapter 0
Rotman, J. Abstract Modern Algebra
Dummit, D. and R. Foote Abstract Algebraic
Meyer, C. Matrix Analysis and Applied Linear Algebraic
Strang, G. Linear Algebra, Learning from Data

Basic Topics (MAS 5145, MAS 5311)

Categories:
Basic definitions and concepts; products, co-products; Universal Properties.

Groups:
Basic definitions, subgroups, normal subgroups, quotient groups; homomorphisms, isomorphism theorems; free groups, free product of groups; permutation groups; Lagrange and Cayleys theorems; actions of groups on sets; Sylows theorems; classification of finite Abelian groups; composition series, Jordan-Holders theorem, solvability; Products and semi-direct products of groups.

Rings:
Basic definitions, ideals, quotient rings; Fields, prime and maximal ideals; Ring homomorphisms, isomorphsm theorems; Free algebras and polynomial rings; Rings of fractions; Noetherian rings, Hilbert Basis Theorem; Irreducibility, UFDs, Gauss Lemma; Euclidean Domains, PIDs; Chinese Remainder Theorem.

Modules:
Basic definitions, submodules, quotient modules; Homomorphisms, isomorphism theorems; Free modules and presentations of modules; Noetherian modules; Classification of finitely generated modules over a PID.

Linear Algebra:
Basic definitions about vector spaces; Subspaces, quotient spaces; Bases of vector spaces; Homomorphisms between vector spaces (linear maps), isomorphism theorems, matrix representation of linear maps; Rank and deficiency of a linear map; Rank and nullity of a matrix, trace and determinant of a matrix and of a linear map; Determinants, non-singular matrices and isomorphisms of vector spaces; Eigenvectors, eigenvalues and diagonalizable matrices; Minimal and characteristic polynomials; Cayley-Hamilton theorem; Rational and Jordan canonical forms; Inner products, orthonormal bases, Gram-Schmidt orthogonalization; Orthogonal and unitary matrices; Symmetric and Hermitian matrices; Spectral Theorem; SVDs; Generalized inverse of a matrix (Moore-Penrose pseudoinverse); Schur factorization.