

PhD Qualifying Exam - Mathematical Statistics Syllabus

Textbook

An Introduction to Probability and Statistics, V.K. Rohatgi and A.K. Md. Ehsanes Saleh, Third Edition, 2015, John Wiley & Sons (ISBN: 978-1-118-79964-2)

Topics

About 50% on Mathematical Statistics I (up to **Limit Theorems**) and 50% on Mathematical Statistics II

Probability

Statistical experiments, Events, Sample Space, Probability Axioms, Combinatorics: Probability on Finite Sample Spaces, Conditional Probability and Bayes Theorem, Independence of Events.

Random Variables and Their Probability Distributions

Random Variables, Probability Distribution of a Random Variable, Discrete and Continuous Random Variables, Functions of a Random Variable. Discrete distributions: Binomial, Hyper Geometric, Poisson, Geometric, Negative Binomial. Continuous distributions: Normal, Gamma, Beta, Weibull, Rayleigh, Log-normal, Logistic.

Moments and Moment Generating Functions

Moments of a Distribution Function, Generating Functions, Some Moment Inequalities.

Multiple Random Variables

Multiple Random Variables, Independent Random Variables, covariance, Correlation, and Moments, Conditional Expectation, Order Statistics and Their Distributions.

Limit Theorems

Modes of Convergence, Weak Law of Large Numbers, Strong Law of Large Numbers, Limiting Moment Generating Functions, Central Limit Theorem.

Sample Moments and Their Distributions

Random Sampling, Sample characteristics and Their distributions, Chi-Square, t-, and F Distributions (Exact Sampling Distributions), Large-Sample Theory, Distribution of Sample Mean and Sample Variance in Sampling from a Normal Population, Sampling from a Bivariate Normal.

Parametric Point Estimation

Problem of Point Estimation, Sufficiency, Completeness, and Ancillarity, Unbiased estimation, Unbiased Estimation: Lower Bound for the Variance of an Estimator, MVUE, Substitution Principle (Method of Moments), Maximum Likelihood Estimators, Bayes and Minimax Estimation, Principle of Equivariance.

Neyman-Pearson theory of testing Hypotheses

Some Fundamental Notions of Hypothesis testing, Neyman-Pearson Lemma, Families with Monotone Likelihood Ratio, Unbiased and Invariant Tests, Locally Most Powerful Tests.

Some Further Results of Hypothesis Testing

Generalized Likelihood Ratio Tests, Chi-Square Tests, t-tests, F-Tests, Bayes and Minimax Procedures.

Confidence Estimation

Some Fundamental Notions of confidence Estimation, Methods of finding confidence Intervals, Shortest-Length confidence Intervals, Unbiased and Equivariant confidence Intervals.