MAD 1100 – Mathematics for Information Technology
SYLLABUS
FALL 2018
Dr. Susan Gorman

Text: A First Course in Discrete Mathematics by John C. Molluzzo and Fred Buckley, (Waveland Press, Inc. 1986).

Mathematics for Informational Technology is an introductory course in discrete mathematics providing the student with a core of mathematical terminology and concepts. The course is not intended to teach the student how to construct mathematical proofs, although the material will require the student to use abstract thinking.

1, Sets and Logic – 3 weeks
   2.1 - 2.8, Use supplement- predicate logic, universal and existential Quantification (see lecture, problems and solutions below)

2. Permutations, Combinations 1-2 weeks
   3.2-3.3

3. Probability - 3-4 weeks
   4.1-4.3, Supplement – Bayes Theorem (see lecture, problems and solutions below)

4. Statistics – 1 week
   4.4 (Exclude sums of random variables)

4 Relations and Functions – 3-4 weeks
   5.1 – 5.5

5. Boolean Algebra – 2 weeks
   7.1 – 7.4

6. Graph Theory – 2 weeks
   8.1, 8.2, 8.3*, 8.4*, 8.5*, 8.6*
   * include if time permits

Predicates

Statements like "$x>3$" are found in mathematical assertions and computer programs. The variable "x" is the subject of the statement. The second part, " is greater than 3" is called the predicate. We denote the statement "x greater than 3 by $P(x)$. The statement $P(x)$ is a propositional function. When a value is assigned to x, $P(x)$ is true or false.

EX. $P(4)=T$, $P(2)=F$

We can also have statements with more than one variable. "$x=y+3$" is $Q(x,y)$, where Q is the predicate, and x and y are the variables.

EX. $Q(1,2)=T$, $Q(3,0)=T$, $Q(5,6)= F$

Quantifiers

Def. The universal quantification of $P(x)$ is "$P(x)$ for all values of x in the domain written $\forall xP(x)$, read "for all x, $P(x)$" or "for every x, $P(x)$".

EX. Let $P(x)$ be "$x+1>x$", Domain – integers

$\forall xP(x) = T$

Let $Q(x)$ be "$x>2$", Domain – integers

$\forall x\, Q(x) = F$ ($Q(1)=F$)

Def. The existential quantifier of $P(x)$ is "there exists an element x in the domain such that $P(x)$", written $\exists xP(x)$

EX. Let $P(x)$ be "$x>3$" Domain – Integers

$\exists xP(x) = T$ (since $P(4)=T$)

Let $Q(x)$ be "$x+1=x$" Domain – integers

$\exists xQ(x) = F$ (no integer satisfies equation)

Negating Quantified Expressions

Negation	Equivalent Statement	
$\sim\exists xP(x)$	$\forall x\sim P(x)$	(For every x, $P(x)$ is false)
$\sim\forall xP(x)$	$\exists x\sim P(x)$	(There is an x for which $P(x)$ is false)

EX. Write the negations of the following statements:

1. There is an honest politician
   $H(x)$: x is an honest politician (Domain – politicians)
   In symbols: $\exists x H(x)$
   To negate: $\sim\exists x H(x) = \forall x \sim H(x)$
   " All politicians are not honest."
2. All Americans eat cheeseburgers
   $C(x)$: x eats cheeseburgers (Domain – Americans)
   To negate: $\sim\forall x C(x) = \exists x \sim C(x)$
   Some American does not eat cheeseburgers.
3. Every student in this class has visited Mexico or Canada.
   $M(x)$: x visited Mexico, $C(x)$: x visited Canada (Domain – students in class)
   $\forall x\, (M(x) \vee C(x))$
   To negate: $\sim \forall x(M(x) \vee C(x)) \equiv \exists x \sim(M(x) \vee C(x)) = \exists x(\sim M(x) \wedge \sim C(x))$ (De Morgans Law)
   There is a student who has not visited Mexico and has not visited Canada. $\vee$ $\vee$

Supplementary Problems – Predicates

1. Let $P(x)$ be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.
   a) $\exists x P(x)$ b) $\forall x P(x)$ c) $\exists x \sim P(x)$ d) $\forall x \sim P(x)$
2. Translate these statements into English, where $C(x)$ is "x is a comedian" and $F(x)$ is "x is funny" and the domain consists of all people.
   a) $\forall x(C(x) \rightarrow F(x))$ b) $\forall x(C(x) \wedge F(x))$ c) $\exists x(C(x) \rightarrow F(x))$ d) $\exists x(C(x) \wedge F(x))$
3. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
   a) No one is perfect.
   b) Not everyone is perfect.
   c) At least one of your friends is perfect.
   d) Everyone is your friend and perfect.
   e) Not everyone is your friend or someone is not perfect.
4. Rewrite so that the negation sign is next to the propositional function.
   a) $\sim\forall x P(x)$ b) $\sim\exists x Q(x)$ c) $\sim\forall x(P(x) \wedge \sim Q(x))$ d) $\sim\exists x(P(x) \vee Q(x))$
5. Write the negation of the following statements (negation next to propositional function)
   a) There is a student who can speak Hindi and cannot speak French.
   b) All students have learned a programming language or have not taken information technology.

Solutions Supplementary Problems – Predicates

1. a) There is a student who spends more than five hours every weekday in class.

   b) All students spend more than five hours every weekday in class.

   c) There is a student who does not spend more than five hours every weekday in class.

   d) All students do not spend more than five hours every weekday in class.

2. a) All people who are comedians are funny.

   b) All people are comedians and are funny.

   c) Some people who are comedians are funny.

   d) There is a person who is a comedian and is funny.

3. Let P(x): x is perfect . Let F(x): x is your friend

   a) $\forall x \sim P(x)$ b) $\sim \forall x P(x)$ c) $\exists x(F(x) \wedge P(x))$ d) $\forall x(F(x) \wedge P(x))$ e) $\sim \forall x F(x) \vee \exists x \sim P(x)$

4. a) $\exists x \sim P(x)$ b) $\forall x \sim Q(x)$ c) $\exists x (\sim P(x) \vee Q(x))$ d) $\forall x(\sim P(x) \wedge \sim Q(x))$

5. a) All students cannot speak Hindi or can speak French

   b) Some students have not learned a programming language and have taken information technology.

Baye's Formula (Supplement)

theorem. $P(F/E) = \dfrac{P(F) \cdot P(E/F)}{P(F) \cdot P(E/F) + P(F')P(E/F')}$

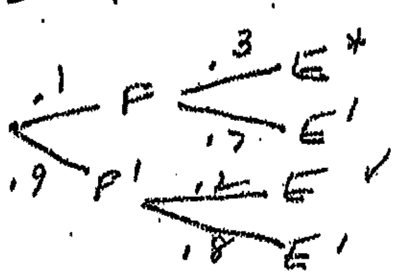
Way of getting $P(F/E)$ by knowing $P(E/F)$
DON'T MEMORIZE

EX The probability of a worker error is .1; probability of an accident when there is a worker error is .3; probability of an accident when no worker error is .2. Find the probability of a worker error if there is an accident.

let E = accident Let F = worker error

$P(F) = .1$ $P(E/F) = .3$ $P(E/F') = .2$

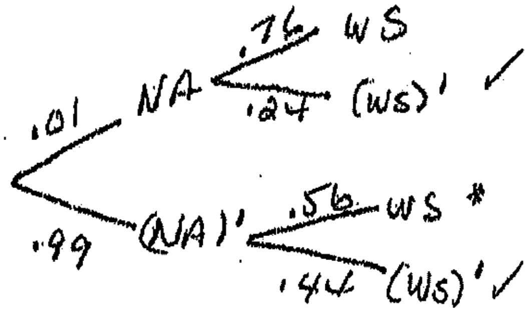
Use a tree:



$P(F/E) = \dfrac{P(E \cap F)}{P(E)}$ ← * first branch; $P(E)$ ← sum of 2 branches ending in E (✓)

$P(F|E) = \dfrac{(.1)(.3)}{(.1)(.3) + (.9)(.2)} = .143$

EX In 2000, 1% of US population was native American (NA). The probability that a native American lives in west or south (WS) is .76. The probability someone not a native American lives in west or south is .56. Find the probability a person who does not live in west or south is not a native American.



$P((NA)'/(WS)') = \dfrac{P((NA)' \cap (WS)')}{P((WS)')}$ ← * branch; ✓ sum of 2 branches ending in (WS)'

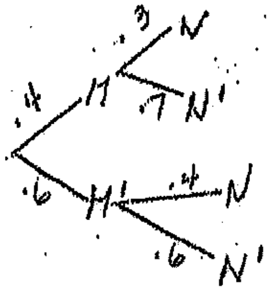
$= \dfrac{(.99)(.44)}{(.99)(.44) + (.01)(.24)} = .9945$

Supplement Problems Bayes Theorem

1. For two events M and N, P(M)=.4, P(N/M) = .3, and P(N/M')=.4. Find each of the following:
   a) P(M/N) b) P(M'/N)
2. A study showed that 68% of all vehicle crashes produced property damage only (as opposed to injuries or fatalities). Of the property-damage-only crashes, 11.0% occurred during rain and 3.2% occurred during snow. For the injury-fatality crashes (not property damage only), 10.5% occurred during rain and 2.2% occurred during snow.
   a) Find the probability that a randomly chosen crash that occurred during snow resulted in property damage only.
   b) Find the probability that a randomly chosen crash that occurred during rain was not property damage only.
3. In 2008, 26.2% of Americans were college graduat4es. The probability that a college graduate used direct deposit with his or her financial institution was .78. For non-college graduat4es, the probability was .62. Suppose a person used direct deposit. Find the probabilities that a person
   a) is a college graduate
   b) is not a college graduate

Solutions Supplement Problems Bayes Theorem

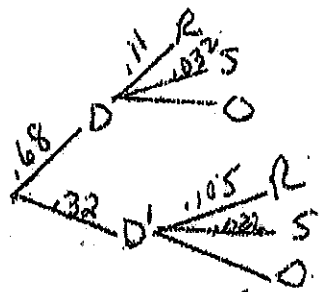
1.



$$P(M/N) = \frac{P(M \cap N)}{P(N)} = \frac{.4(.3)}{.4(.3) + .6(.4)} = \frac{.12}{.12 + .24} = \frac{12}{36} = \frac{1}{3}$$

$$P(M'/N) = \frac{P(M' \cap N)}{P(N)} = \frac{.6(.4)}{.4(.3) + .6(.4)} = \frac{2}{3}$$

2.

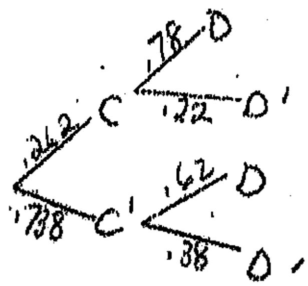


D = property damage only R = rain S = snow
O = other conditions.

$$P(D/S) = \frac{P(D \cap S)}{P(S)} = \frac{.68(.032)}{.68(.032) + .32(.022)} = .76$$

$$P(D'/R) = \frac{P(D' \cap R)}{P(R)} = \frac{(.32)(.105)}{.68(.11) + (.32)(.105)} = .31$$

3.



C = college graduate D = use direct deposit

$$P(C/D) = \frac{P(C \cap D)}{P(D)} = \frac{(.262)(.78)}{(.262)(.78) + (.738)(.62)} = .31$$

$$P(C'/D) = \frac{P(C' \cap D)}{P(D)} = \frac{(.738)(.62)}{(.262)(.78) + (.738)(.62)} = .69$$